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Robust virtual implementation under common strong belief in rationality

Christoph Müller

School of Economics, The University of Queensland, Level 6, Colin Clark Building (39), St Lucia, Qld 4072, Australia

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Abstract

Bergemann and Morris (2009b) show that static mechanisms cannot robustly virtually implement non-constant social choice functions if preferences are sufficiently interdependent. Without any knowledge of how agents revise their beliefs this impossibility result extends to dynamic mechanisms. In contrast, we show that if the agents revise their beliefs according to the forward-induction logic embedded in strong rationalizability, admitting dynamic mechanisms leads to considerable gains. We show that all ex-post incentive compatible social choice functions are robustly virtually implementable in private consumption environments satisfying a weak sufficient condition, *regardless* of the level of preference interdependence. This result derives from the key insight that in such environments, in every belief-complete type space under common strong belief in rationality (Battigalli and Siniscalchi, 2002), dynamic mechanisms can distinguish all payoff type profiles by their strategic choices. Notably, dynamic mechanisms can robustly virtually implement the efficient allocation of an object in cases that static mechanisms cannot.

JEL classification: C72; C73; D78; D82

Keywords: Strategic distinguishability; Common strong belief in rationality; Extensive-form rationalizability; Strong rationalizability; Robust implementation; Virtual implementation

E-mail address: c.mueller@uq.edu.au.

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1. Introduction

Through their stringent necessary conditions for implementation, many results from a growing literature on belief-free or robust[1] implementation reveal an unwelcome trade-off for mechanism designers: Either choose one of the comparatively few social choice functions that are robustly implementable, or give up robustness. The former choice is severely limiting in many environments and sometimes confines designers to choose among constant social choice functions. The latter choice entails reverting to non-robust mechanisms, which depend on fine details about the agents' assumed beliefs and higher-order beliefs about private information. Results by Bergemann and Morris (2009b) (henceforth BM) establish this trade-off for robust virtual implementation by static mechanisms. In this paper, we show that if designers use dynamic mechanisms and adopt a natural generalization of BM's notion of robustness, then they can enlarge the set of robustly virtually implementable social choice functions and escape the described trade-off in important cases.

Assumptions about agents' beliefs and higher-order beliefs about payoff types are usually described by a type space, often implicitly via the assumption of a common prior over a payoff type space. Motivated by the desire to avoid such assumptions, BM define *robust virtual implementation* (rv-implementation) as (full) rationalizable virtual implementation. This is justified by the well-known equality between the union of all Bayesian Nash equilibrium strategies across all type spaces on the one hand, and the set of rationalizable strategies on the other hand (Brandenburger and Dekel, 1987; Battigalli and Siniscalchi, 2003). If a social choice function is rationalizably implementable, then it is implementable in Bayesian Nash equilibrium for all type spaces, freeing the designer from the task of guessing the "correct" type space.[2]

Instead of anchoring the concept of robust implementation in an equilibrium concept, a second justification for BM's definition formulates the notion of implementation directly in terms of explicit assumptions about the agents' strategic reasoning. This justification derives from the characterization of rationalizability by the epistemic condition of rationality and common belief in rationality (RCBR). Just as the equilibrium based approach, implementation with respect to RCBR frees the designer from guessing the correct type space, as for every type space, every strategy that is consistent with "RCBR in the type space" is rationalizable.[3] And since RCBR itself does not impose any restrictions on belief hierarchies about payoff types, implementation with respect to RCBR can justly be called robust.

The approach of examining implementation with respect to an epistemic condition can be extended to dynamic mechanisms.[4] Generalizing from above, it is natural to assume that agents

[1] We use the terms "robust" and "belief-free" interchangeably. Some less demanding notions of robustness pursued in the literature are discussed in Subsection 1.1.

[2] Epistemic results are often stated in terms of epistemic type spaces describing interactive beliefs about payoff types *and* strategies (see e.g. Battigalli and Siniscalchi, 1999). In this introduction we only consider epistemic type spaces that express belief restrictions that can also be captured by type spaces about (initial) beliefs about payoff types, and tacitly rule out all other epistemic type spaces. We simply write "type space" even if we refer to an "epistemic type space."

[3] BM adopt a special case of Δ-rationalizability (Battigalli, 1999, 2003; Battigalli and Siniscalchi, 2003) called belief-free rationalizability. A strategy is consistent with RCBR precisely if it is belief-free rationalizable (see Battigalli et al., 2011, and references therein). Moreover, a strategy is consistent with the restrictions on belief hierarchies about payoff types expressed by a given type space and RCBR if and only if it is interim correlated rationalizable for that type space (Dekel et al., 2007; Battigalli et al., 2011). Therefore, every strategy that is interim correlated rationalizable for some type space is also belief-free rationalizable.

[4] Alternatively, one can generalize the equilibrium based approach. Viable generalizations emerge from the equivalence of the set of weak perfect Bayesian equilibrium strategies to the set of weakly rationalizable strategies