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Notes

Finite-population evolution with rare mutations in asymmetric games ☆

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Abstract

We model evolution according to an asymmetric game as occurring in multiple finite populations, one for each role in the game, and study the effect of subjecting individuals to stochastic strategy mutations. We show that, when these mutations occur sufficiently infrequently, the dynamics over all population states simplify to an ergodic Markov chain over just the pure population states (where each population is monomorphic). This makes calculation of the stationary distribution computationally feasible. The transition probabilities of this embedded Markov chain involve fixation probabilities of mutants in single populations. The asymmetry of the underlying game leads to fixation probabilities that are derived from frequency-independent selection, in contrast to the analogous single-population symmetric-game case (Fudenberg and Imhof, 2006). This frequency independence is useful in that it allows us to employ results from the population genetics literature to calculate the stationary distribution of the evolutionary process, giving sharper, and sometimes even analytic, results. We demonstrate the utility of this approach by applying it to a battle-of-the-sexes game, a Crawford–Sobel signalling game, and the beer-quiche game of Cho and Kreps (1987).

JEL classification: C62; C72; C73

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1. Introduction

In evolutionary game theory, games are played within populations, and the prevalence of different strategies changes over time according to natural-selection-like dynamics (Maynard Smith, 1982; Weibull, 1997; Hofbauer and Sigmund, 1998; Samuelson, 1998; Nowak, 2006; Sandholm, 2010). This provides a natural method by which to model biological evolution (Maynard Smith, 1982) and various learning processes (Fudenberg and Levine, 1998), and offers a 'rationality-light' approach to equilibrium selection (Samuelson, 1998).

In the classical approach, populations are infinitely large and dynamics are deterministic; the focus is typically on the equilibrium refinement of evolutionary stability (Maynard Smith, 1982; Hofbauer and Sigmund, 1998). More recently, stochastic finite-population dynamics have been introduced into evolutionary game theory (Foster and Young, 1990; Kandori et al., 1993; Young, 1993; Nowak, 2006; Fudenberg and Imhof, 2006; McAvoy, 2015a). These often take the form of an ergodic Markov chain—for example, when there is a positive mutation rate—the state space of which is all possible strategy compositions of the population (Fudenberg and Imhof, 2006). Ranking the various population states' weights in the stationary distribution is then a natural method of equilibrium selection (Foster and Young, 1990; Kandori et al., 1993), and solves many problems of the deterministic approach.

A drawback is that the state space is often very large, making calculation of the stationary distribution infeasible. Addressing this, Fudenberg and Imhof (2006) study the case of a symmetric game played within a single, finite population, and show that, when the mutation rate is very small, the evolutionary process simplifies significantly. The intuition is straightforward: Starting from a pure (monomorphic) population state, we wait a very long time for a new strategy to appear in the population, because the mutation rate is small. When it does, it either goes extinct or takes over the population ('fixes'). Because this resolution of the mutant's fate occurs on a much shorter timescale than the waiting time for another mutation to occur, it typically re-establishes a pure state. The process therefore approximates a simpler process over just the pure states. This dramatic reduction of the state space makes calculation of the stationary distribution computationally simple.

The transition probabilities of this simpler process depend critically on the various mutants' fixation probabilities—the probability that a given strategy, having arisen in a population otherwise pure for a different strategy, subsequently fixes in that population. Because the game is symmetric, the payoffs that determine these fixation probabilities are frequency dependent—the payoff to a mutant strategy changes as its frequency in the population increases. For most evolutionary processes, frequency-dependent fixation probabilities either do not exist in closed form, or are intractable when they do (Nowak, 2006). This significantly limits the analytical use of Fudenberg and Imhof's result.

Here, we employ the basic machinery of Fudenberg and Imhof (2006) to derive a result similar to theirs for asymmetric games. There are several reasons why such a result is desirable. First, many situations in which we might want to study evolutionary or learning dynamics are best modelled as asymmetric games—for example, signalling games (Spence, 1973; Crawford and Sobel, 1982; Grafen, 1990), games of entry and entry-deterrence (Salop, 1979; Milgrom and Roberts, 1982; Maynard Smith and Parker, 1976), and games of time consistency and commitment (Kydland and Prescott, 1977). Second, because only strict Nash equilibria of asymmetric games are evolutionarily stable (Samuelson and Zhang, 1992), the deterministic approach based on evolutionary stability often fails. This is especially true for multi-stage asymmetric games, which typically have no strict Nash equilibria (because alternative strategies that