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Existence of equilibria in discontinuous Bayesian games ☆

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Abstract

We provide easily-verifiable sufficient conditions on the primitives of a Bayesian game to guarantee the existence of a behavioral-strategy Bayes–Nash equilibrium. We allow players' payoff functions to be discontinuous in actions, and illustrate the usefulness of our results via an example of an all-pay auction with general tie-breaking rules which cannot be handled by extant results.

JEL classification: C62; C72; D82

Keywords: Discontinuous Bayesian game; Behavioral strategy; Random disjoint payoff matching; Equilibrium existence; All-pay auction

1. Introduction

Bayesian games, where each player observes his own private information and then all players choose actions simultaneously, have been extensively studied and found wide applications in many fields of economics. The notion of Bayesian equilibrium is a fundamental game-theoretic concept for analyzing such games. In many applied works, Bayesian games with discontinuous payoffs arise naturally. For example, in auctions or price competitions, players' payoffs may not

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be continuous when a tie occurs. However, many previous works focus on the case of continuous payoffs,[1] while little is known about equilibrium existence results in Bayesian games with payoff discontinuities.

In a complete information environment, Reny (1999) showed that a better-reply secure game possesses a pure-strategy Nash equilibrium, and proposed the payoff security condition which is sufficient for a game to be better-reply secure together with some other conditions.[2] Recently, several authors have generalized the work of Reny (1999) to an incomplete information setting. Specifically, Carbonell-Nicolau and McLean (2015) extended the "uniform payoff security" condition of Monteiro and Page (2007) and the "uniform diagonal security" condition of Prokopovych and Yannelis (2014) to the setting of Bayesian games, and showed the existence of behavioral/distributional-strategy equilibria. He and Yannelis (2015a) proposed the "finite payoff security" condition and proved the existence of pure-strategy equilibria.

The purpose of this paper is to provide a new equilibrium existence result for Bayesian games with discontinuous payoffs. Our result is based on a Bayesian generalization of the clever condition called "disjoint payoff matching", which was introduced by Allison and Lepore (2014) for a normal form game. The advantage of this condition is that one only needs to check the payoff at each strategy profile itself. The standard payoff security-type condition forces one to check the payoffs in the neighborhood of each strategy profile, which is more demanding. Thus, our condition is relatively straightforward, and the equilibrium existence result can be easily verified for a large class of Bayesian games. Our result widens the applications in economics as we can cover situations that previous results in the literature are not readily applicable. As an illustrative example, we provide an application to an all-pay auction with general tie-breaking rules.

The rest of the paper is organized as follows. The model and our main results are presented in Section 2. Some preparatory results and the proof of the main theorem are collected in Section 3. An illustrative application to an all-pay auction with general tie-breaking rules is given in Section 4. Section 5 concludes the paper.

2. Model

2.1. Bayesian game and behavioral-strategy equilibrium

We consider a **Bayesian game** as follows:

$$G = \{u_i, X_i, (T_i, \mathcal{T}_i), \lambda\}_{i \in I}.$$

- There is **a finite set of players**, $I = \{1, 2, \ldots, n\}$.
- Player i's **action space** X_i is a nonempty compact metric space, which is endowed with the Borel σ-algebra $\mathcal{B}(X_i)$. Denote $X = \prod_{i \in I} X_i$ and $\mathcal{B}(X) = \otimes_{i \in I} \mathcal{B}(X_i)$; that is, $\mathcal{B}(X)$ is the product Borel σ-algebra.
- The measurable space $(T_i, \mathcal{T}_i)$ represents the **private information space** of player i. Let $T = \prod_{i \in I} T_i$ and $\mathcal{T} = \otimes_{i \in I} \mathcal{T}_i$.
- The **common prior** λ is a probability measure on the measurable space $(T, \mathcal{T})$.

[1] See, for example, Milgrom and Weber (1985) and Balder (1988).

[2] A number of recent papers have generalized the work of Reny (1999) in several directions; see Bagh and Jofre (2006), Carmona (2009), Bagh (2010), Carbonell-Nicolau and McLean (2013), Prokopovych (2013), Reny (2015), Carmona and Podczeck (2014, 2015), and He and Yannelis (2015b) among others.