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Journal of Economic Theory 160 (2015) 1–23

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Limits to rational learning

Yehuda John Levy [1]

Nuffield College and Department of Economics, University of Oxford, United Kingdom

Received 11 November 2014; final version received 11 June 2015; accepted 2 August 2015

Available online 17 August 2015

Abstract

A long-standing open question raised in the seminal paper of Kalai and Lehrer (1993) is whether or not the play of a repeated game, in the rational learning model introduced there, must eventually resemble the play of *exact* equilibria, and not just the play of *approximate* equilibria as demonstrated there. This paper shows that play may remain distant – in fact, mutually singular – from the play of any equilibrium of the repeated game. We further show that the same inaccessibility holds in Bayesian games, where the play of a Bayesian equilibrium may continue to remain distant from the play of any equilibrium of the true game.

JEL classification: C73; D83; C65

Keywords: Rational learning; Repeated games; Nash equilibrium

1. Introduction

The premise of *rational learning* is that decision-making agents update their beliefs about what other agents will do based on the actions that they have observed. The seminal work of Blackwell and Dubins (1962) shows that when a single agent learns rationally in this way, even if his prior beliefs about the process are incorrect but do contain a minimal amount of truth, his posterior beliefs will eventually lead to true beliefs about the process. The staple work in game theory which incorporates this paradigm into the multi-agent setting is Kalai and Lehrer (1993),

E-mail address: john.calculus@gmail.com.

[1] The author expresses thanks to Peyton Young for his helpful remarks and for bringing to the author's attention the conjecture this paper addresses, and to Ron Peretz for his help with the probabilistic tools this paper utilises.

http://dx.doi.org/10.1016/j.jet.2015.08.001

in which agents both learn and also try to maximise their utility in a repeated game. Their work studies the question of whether the beliefs of the agents not only merge, but converge to beliefs induced by the *Nash equilibria* of the repeated game. As it turns out, the answer is, not necessarily.

Kalai and Lehrer (1993) was not the only work at that time to explore questions of convergence under rational learning; in fact, other influential works that both support it and contrast with it were carried out around the same time and in the following years. Jordan (1991), for example, shows that under appropriate assumptions, rational learning converges to the set of stage game Nash equilibria. However, Kalai and Lehrer (1993) struck on a property of learning discussed very little at the time: the convergence of the agents' strategies to (approximate) equilibria of the *repeated game*, not of the strategic game that is being repeated. While their assumptions that the players' beliefs all contain a grain of truth[2] have been scrutinised as both an over-demanding coordination requirement (e.g., Miller and Sanchirico, 1999) and a highly non-generic condition (e.g., Miller and Sanchirico, 1997), the resulting body of literature extending, discussing, and contrasting (Kalai and Lehrer, 1993) "is, in many respects, a natural successor to the earlier literature on learning rational expectations" as "both literatures address the question of whether decision-makers can, through repeated experience, learn to make optimal or equilibrium decisions" (Jordan, 1993).

A primary and motivating example in which such learning occurs naturally is the class of *Bayesian games*, in particular when the preferences of the agents – that is, their *types* – are not known publicly but others do have beliefs about them, which are updated at each stage. Such interactions occur naturally, for example, in sequential auctions, where the private values of the object being sold are not commonly known; however, agents learn more about the others' preferences as time goes on and bids are observed, e.g. Jeitschko (1998). Sequential bargaining with incomplete information, wars of attrition, and repeated duopolistic competition when others' costs are uncertain all naturally fit under this umbrella framework as well.

Two main veins of subsequent work exist. One direction generalises the results of Kalai and Lehrer (1993), some works by weakening the absolute continuity assumptions on the beliefs, as in Sandroni (1998) and Norman (2012), others by weakening assumptions on the players' knowledge, as in Kalai and Lehrer (1995), Jordan (1995), and Nyarko (1998), and still other variations, e.g., Gilli (2001) and the references there. Another direction, however, was to point out the limitations of the assumptions and results, as in the papers (Lehrer and Smorodinsky, 1996; Lehrer and Smorodinsky, 1997; Miller and Sanchirico 1997, 1999; Nachbar, 1997, and Foster and Young, 2001).

The contribution of this paper is to answer a long-standing open question raised in Kalai and Lehrer (1993, Sec. 7.1). The classical result of that paper ensures convergence of the play to the set of *approximate* equilibria (i.e., ε-equilibria) of the repeated game. (One cannot in general expect convergence to a specific equilibrium or approximate equilibrium, as players may, for example, rotate among different equilibria.) The authors raise, but leave open, the question of whether play must converge to the set of *exact* equilibria. In this paper we show by example that this need not be the case. Furthermore, not only does convergence fail to occur, but the play induced by any Nash equilibrium remains far – in fact, mutually singular – from actual play.

Kalai and Lehrer (1993, Sec. 6) address in particular the question of rational learning in certain Bayesian games, which, as mentioned previously, are a primary and general class of such

[2] Or somewhat weaker absolute continuity conditions.