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# Priority classes and weighted constrained equal awards rules for the claims problem

Karol Flores-Szwagrzak

*University of Southern Denmark, Denmark*

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**Abstract**

We revisit the "claims problem" (O'Neill, 1982), where a group of individuals have claims on a resource but there is not enough of it to honor all of the claims. We characterize the rules satisfying three well-known invariance axioms: consistency, composition up, and claims truncation invariance. They are priority-augmented versions of the standard weighted constrained equal awards rules, also known as weighted gains methods (Moulin, 2000): individuals are sorted into priority classes; the resource is distributed among the individuals in the first priority class using a weighted constrained equal awards rule; if some of the resource is left over, then it is distributed among the individuals in the second priority class, again using a weighted constrained equal awards rule; the distribution carries on in this way until the resource is exhausted. Our characterization extends to a generalized version of the claims problem where there are multiple divisible and indivisible resources and individuals have claims on each of these.


*JEL classification:* D70; D63; D71

*Keywords:* Consistency; Composition up; Claims truncation invariance

## 1. Introduction

We revisit the "claims problem" (O'Neill, 1982), where a group of individuals have claims on a resource but there is not enough of it to honor all of the claims. An example is the distribution

*E-mail address:* karolszw@sam.sdu.dk.

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of the liquidation value of a bankrupt firm among its creditors. Alternatively, we can think of the distribution of a tax burden among individuals with different taxable incomes (Young, 1987, 1988).

Our goal is to evaluate rules – distribution methods – for the claims problem on the basis of three invariance properties:

- *Consistency* (Aumann and Maschler, 1985; Young, 1987) specifies that if a distribution is considered desirable for some group of individuals, then it should be considered desirable when restricted to each subgroup.[1]
- *Composition up* (Young, 1988; Moulin, 1987) specifies that, upon an increase in the endowment, the rule can recommend the distribution in two equivalent ways: (i) Apply the rule directly to distribute the larger endowment. (ii) Apply the rule to distribute the initial endowment and, thereafter, apply it again to allocate the increment according to the outstanding claims.[2]
- *Claims truncation invariance* (Curiel et al., 1987; Dagan and Volij, 1993) specifies that the excess of a claim over the endowment should be omitted from consideration.[3] As expressed by Aumann and Maschler (1985) in the bankruptcy context, "any amount of debt to one person that goes beyond the entire estate might well be considered irrelevant; you cannot get more than there is".

All three properties are the subject of a growing literature (for surveys, see Thomson, 2003, 2015a; Moulin, 2002).[4]

We characterize the family of rules satisfying consistency, composition up, and claims truncation invariance. The "constrained equal awards rule" already discussed in the 12th century by Maimonides (Aumann and Maschler, 1985) is the rule in this family that equalizes awards subject to no one receiving more than her claim. The weighted generalizations of this rule, the "weighted constrained equal awards rules" also known as "weighted gains methods" (Moulin, 2000), are also members of the family.

In fact, each rule in the family is a priority-augmented weighted constrained equal awards (PWCEA) rule: individuals are sorted into priority classes; the resource is first distributed among the individuals in the first priority class using a weighted constrained equal awards rule; if some of the resource is left over after fully serving the first priority class, it is distributed among the individuals in the second priority class, again using a weighted constrained equal awards rule; the distribution carries on in this way until the resource is exhausted.

The PWCEA rules are members of the family of rules characterized by Moulin (2000) on the basis of consistency, composition up, "composition down", and "homogeneity".[5] The PWCEA rules do not include some rules in Moulin's family such as the "proportional rule" or the "constrained equal losses rule" and its weighted generalizations. Our characterization thus determines

[1] Within claims problems, consistency has been a central property in the study of families of rules (Young, 1987, 1988; Moulin, 2000; Hokari and Thomson, 2003; Chambers, 2006; Kaminski, 2006; Thomson, 2007, 2008, 2015b, 2015c; Stovall, 2014a, 2014b; Chambers and Moreno-Ternero, 2015).

[2] Moulin (1987) refers to the property as "path independence".

[3] Dagan and Volij (1993) and Dagan (1996) refer to the property as "independence of irrelevant claims".

[4] The three properties also reflect Talmudic principles; see Aumann and Maschler (1985) for consistency and Dagan (1996) for composition up and claims truncation invariance.

[5] Moulin (2000) refers to composition up as "lower composition" and to composition down as "upper composition".