Available online at www.sciencedirect.com
ScienceDirect
Journal of Economic Theory 160 (2015) 114–131
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Implementability under monotonic transformations in differences ☆

Juan Carlos Carbajal [a,*], Rudolf Müller [b]

[a] *School of Economics, The University of New South Wales, Sydney, NSW 2052, Australia*
[b] *School of Economics and Business, Maastricht University, Maastricht, The Netherlands*

Received 20 January 2015; final version received 3 September 2015; accepted 4 September 2015

Available online 10 September 2015

Abstract

In a social choice setting with quasilinear preferences and monetary transfers, a domain D of admissible valuations is called a *monotonicity domain* if every 2-cycle monotone allocation rule is truthfully implementable (in dominant strategies). D is called a *revenue equivalence domain* if every implementable allocation rule satisfies the revenue equivalence property. We introduce the notions of *monotonic transformations in differences*, which can be interpreted as extensions of Maskin's monotonic transformations to quasilinear environments, and show that if D admits these transformations then it is a monotonicity and revenue equivalence domain. Our proofs are elementary and do not rely on strenuous additional machinery. We illustrate monotonic transformations in differences for settings with finite and infinite allocation sets.

JEL classification: C72; D70; D82

Keywords: 2-Cycle monotonicity; Cyclic monotonicity; Truthful implementability; Revenue equivalence; Monotonic transformations in differences

☆ A previous version circulated as *Plasticity, Monotonicity, and Implementability*. We would like to thank the very helpful comments by the Associate Editor and the referees. Many thanks to Debasis Mishra, Claudio Mezzetti, Andrew McLennan, Wojciech Olszewski, Jean-Charles Rochet, Arunava Sen, Rabee Tourky, Rakesh Vohra and John Weymark, and the audiences at the 2014 SSCW Meeting in Boston, the 2013 CMSS Workshop in Auckland, and the 2013 Dagstuhl Workshop on Electronic Markets and Auctions. The first author gratefully acknowledges financial support from the Australian Research Council under grant DE120102640.

* Corresponding author.
E-mail addresses: jc.carbajal@unsw.edu.au (J.C. Carbajal), r.muller@maastrichtuniversity.nl (R. Müller).

http://dx.doi.org/10.1016/j.jet.2015.09.001
0022-0531/

1. Introduction

We consider allocation problems in a mechanism design setting with quasilinear utilities over monetary transfers and social allocations. A valuation for an agent, which fully captures his relative preferences over alternatives, is a real function defined on the allocation set and is the agent's private information. We treat the set of admissible valuations – the *preference domain* – as our primitive. We study truthful (dominant strategy) implementability of direct revelation mechanisms composed of an allocation rule mapping admissible valuations onto the allocation set, and an additional payment rule mapping profiles of valuations into monetary transfers. Our purpose is to contribute to the mechanism design program in its identification of settings where *well-behaved mechanisms* exist – i.e., mechanisms in which the allocation rule can be said to be truthfully implementable if it satisfies a system of constraints independent of payments, and for which any incentive compatible payment rule is expressible in terms of the allocation rule alone.

One of the earliest contributions comes from Myerson's (1981) auction design work, which shows that in single parameter settings one can replace the incentive constraints with a simple monotonicity requirement on the allocation rule. Once obtained, the monotone allocation rule could be used to completely recover the incentive compatible payments (up to a constant). Myerson's monotonicity condition does not extend easily to multi-dimensional settings. However, it has been known since Rochet (1987) that a *cyclic monotonicity* condition on an allocation rule is equivalent to its truthful implementability in every quasilinear environment. In other words, Rochet showed that an allocation rule is truthfully implementable if, and only if, its corresponding allocation graph contains no cycle of negative length.

More recently, Bikhchandani (2006), Saks and Yu (2005), and Archer and Kleinberg (2014) showed that the weaker *2-cycle monotonicity* is not only necessary but also sufficient for truthful implementability in convex domains with a finite allocation set. Mishra et al. (2014) developed a similar result on non-convex domains based on ordinal properties, still working with a finite allocation set. Our contribution is finding a *new* set of sufficient attributes on the preference domain to ensure that 2-cycle monotonicity is not only necessary but also sufficient for implementation. What distinguishes our work from previous results in the literature is that our conditions apply to *finite and infinite* allocation sets. As it turns out, these attributes also guarantee that every implementable allocation rule satisfies revenue equivalence.

Section 2 contains the details of our model. For notational convenience, we deal with a single-agent environment. A preference domain D is a subset of the space of real-valued functions defined on the allocation set A. We impose no a priori restriction on the cardinality of A. The domain D is called a *monotonicity domain* if every 2-cycle monotone allocation rule is truthfully implementable (in dominant strategies). It is called a *revenue equivalence domain* if every implementable allocation rule satisfies revenue equivalence. In Section 3 we establish a convenient representation of truthful implementability for allocation rules whose 2-cycles have zero length. Indeed, if the sum of the lengths between any pair of alternatives defined by a 2-cycle monotone allocation rule is exactly zero, then the allocation rule is 3-cycle monotone if, and only if, it is cyclically monotone. This follows because one can transform the sum of the lengths in a 4-cycle to an equivalent sum of lengths of two adjacent 3-cycles, which have non-negative length. Applying this argument recursively yields the desired conclusion.

In Section 4 we introduce the notion of *monotonic transformations in differences*, which can be considered as adaptations to the quasilinear environment of Maskin's (1999) monotonic transformations, which have been extensively employed in the social choice literature. To help intuition, we include simple examples of domains that admit monotonic transformations in dif-