Available online at www.sciencedirect.com

ScienceDirect

Journal of Economic Theory 157 (2015) 28–48

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Public information in Markov games

Andrew Kloosterman [1]

University of Virginia, Department of Economics, P.O. Box 400182, Charlottesville, VA 22904-4182, United States

Received 11 April 2013; final version received 29 October 2014; accepted 28 November 2014

Available online 10 December 2014

Abstract

In a Markov game, players engage in a sequence of games determined by a Markov process. In this setting, this paper investigates the impact of varying the informativeness of public information, as defined by Blackwell [8,9], pertaining to the games that will be played in future periods. In brief, when a curvature condition on payoffs is satisfied, the finding is that, for any fixed discount factor, the set of strongly symmetric subgame perfect equilibrium payoffs of a Markov game with more informative signals is contained in this set of equilibrium payoffs if the Markov game is played with any less informative signals. The second result shows that larger equilibrium payoffs are possible with less informative signals when the curvature condition fails, but only for some discount factors. The third result strengthens the curvature condition, but generalizes the first result to all subgame perfect equilibrium payoffs. Finally, a collusion application is presented to illustrate the curvature condition.

JEL classification: C73

Keywords: Markov games; Value of information; Collusion

E-mail address: ask5b@virginia.edu.

[1] I am especially grateful to my advisors Ennio Stacchetti and David Pearce for their help on this project. I also wish to thank Tomasz Sadzik, Dilip Abreu, Laurent Mathevet, Jack Fanning, Elliot Lipnowski, Ilari Paasivirta, Peter Troyan and everyone else who has discussed it with me for many insightful comments.

http://dx.doi.org/10.1016/j.jet.2014.11.018
0022-0531/

1. Introduction

A well-known fact of dynamic games is that intertemporal incentives can support equilibrium outcomes that are not possible in static games; threats of punishment in the future induce players to choose actions that are not myopically optimal. Many dynamic environments feature uncertainty about the nature of future interactions. As the future determines what punishments are possible, it is reasonable to suppose that information regarding it will impact equilibrium outcomes. However, it is unclear what this impact will be, and investigating this question forms the focus of this paper.

Dynamic environments with uncertainty about the future are ubiquitous in economics. Consider the example of collusion over the business cycle (Rotemberg and Saloner [35], Haltiwanger and Harrington Jr. [19], Kandori [22], Bagwell and Staiger [5]). There is an infinite horizon over which an oligopoly of firms would like to collude, but, unlike the standard repeated oligopoly problem, demand fluctuates over time as the economy goes through booms and busts. Examples of information that firms may possess regarding future demand include industry demand reports and macroeconomic indicators. This paper speaks to whether it is high or low quality information that makes collusion easier. As shall be seen, the answer turns out to be low quality information.

Formally, the environment is a game-theoretic setting called Markov games (also commonly called stochastic games). Markov games generalize infinitely repeated games by allowing the game played each period to vary. There is a set of possible games and one is selected each period according to a Markov process. That is, the game in each period is randomly drawn from a distribution that depends on the game and potentially, although not in this paper, the players' actions in the previous period.

In this paper, the Markov games model is modified with the addition of public information. Specifically, in every period, before they choose their actions, the players receive a public signal about the game in the next period.[2] Because the current game is known, there are no direct payoff consequences of the signals. However, as mentioned previously, information is relevant for equilibrium behavior because beliefs about future games determine which intertemporal incentives can be supported. For example, returning to the collusion game, a signal that predicts high demand in the next period would make collusion easier, as a cheating firm would forgo more expected future profit.

Employing Blackwell's [8,9] criterion to define how one set of signals is more informative than another, this paper compares the equilibrium payoffs of two hypothetical worlds where the same Markov game is played but with more informative public signals in one world, called world M, than in the other world, called world L.

There are three results. First, under a curvature condition on payoffs, the set of strongly symmetric subgame perfect equilibrium payoffs in world M is contained in this set of equilibrium payoffs in world L. The result holds for any fixed discount factor. In the sense that larger equilibrium payoffs are possible in world L, the value of information is negative. Second, when no curvature condition is imposed, there are still some discount factors for which the best feasible payoff in the Markov game is an equilibrium payoff only in world L. Therefore, the value of information is still negative for these discount factors. Finally, the first result is generalized to the

[2] The game in the following period must be known by some agent outside the Markov game (usually nature) that generates the public signal. As signals arrive before actions are taken, this is why the Markov process cannot depend on the actions of the players in the previous period.