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# Correlation and common priors in games with incomplete information ☆

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**Abstract**

This paper provides an explicit characterization of correlations that are implicitly captured by partition models for incomplete information games. The main result of this paper shows that every partition model for incomplete information can be decomposed into the conjunctions of a unique non-redundant model and a unique "individually uninformative" correlating device. The separation of the correlating device from the underlying non-redundant model allows us to separate their strategic implications. As an application, we use this correlation device to define correlated equilibria. Separating the common-prior property of the correlating devices from the underlying state space sheds light on the difference between interim independent and correlated rationalizability.


*JEL classification:* C72; D80

*Keywords:* Correlation; Common prior; Correlated equilibrium; Interim rationalizability; Redundant types

## 1. Introduction

Aumann [1] makes the observation that an explicit use of correlating devices can expand the set of equilibrium outcomes. This insight has been further developed in the context of incomplete information games; see, e.g., Forges [11] and Bergemann and Morris [4]. A recent strand of the

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type space literature offers a different perspective on correlation. This literature makes the observation that "redundant" types implicitly capture the correlation of beliefs, and consequently, type spaces with the same set of Mertens–Zamir [18] hierarchies of beliefs can give different predictions; for example, Ely and Peski [9] show exactly what type-space information can characterize interim *independent* rationalizability (IIR); Dekel, Fudenberg, and Morris [7] study interim *correlated* rationalizability (ICR) that is invariant to type space modeling of Mertens–Zamir hierarchies.

In sum, the correlated equilibrium literature models correlation *explicitly* through correlating devices, while the recent type space literature takes an *implicit* approach to correlation. What is the connection between the two approaches to correlation? In this paper, we show how to distill the correlation embedded in type spaces, and characterize the implicit correlation through explicit correlating devices. We then show how this characterization can be applied to study equilibrium properties of incomplete information games and the strategic implications of the common-prior assumption.

We focus on finite partition type spaces for simplicity.[1] We first propose a notion of correlating devices for incomplete information games, which we regard as an analog of Aumann's [1] subjective "roulette." In contrast to Aumann's original complete information framework, the parameters of the correlating devices depend on the payoff and belief states. The correlating device is defined by the property of "individual uninformativeness"—no individual will update the likelihood ratio between any two states of the world upon any private observation. We say a correlating device admits a common prior if players' beliefs over the random outcomes of the correlating device agree state by state. Hence the common-prior property of a correlating device is independent of the common-prior property of the type space on which the correlating device operates.

We want to emphasize that the main focus of this paper is not on redundant types per se; rather, we are interested in the correlation implicitly captured by the redundant types. The main result of this paper shows that every model of incomplete information games can be decomposed into the conjunction of a unique nonredundant model and a unique individually uninformative correlating device. This characterization connects the explicit and implicit approaches to correlation. Hence, it allows us to study incomplete information through correlating devices and study explicit correlation through type spaces with implicit correlation. Moreover, the separation allows us to distinguish the strategic implications of the embedded correlating device from those of the underlying non-redundant type space, thereby shedding new light on the understanding of incomplete information games.

As an application of the main result, we define "correlated equilibria" for incomplete information games with respect to the correlating devices, with or without common priors, as potential extensions of Aumann's correlated equilibrium. Further refinements can be obtained by imposing additional requirements directly on the correlating device. The no-common-prior version of the correlated equilibrium is invariant to type space modelling of Mertens–Zamir beliefs over payoff fundamentals. Both the no-common-prior version of Bayes correlated equilibrium (BCE) introduced by Bergemann and Morris [4] and ICR are invariant. In the no-common-prior case, our correlated equilibrium is finer than BCE but outcome equivalent to ICR. Hence, invariance alone

[1] Please see an early working paper Liu [15] for a treatment of general topological type spaces.