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Minimum cost connection networks: Truth-telling and implementation ☆

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Abstract

In the present paper we consider the allocation of costs in connection networks. Agents have connection demands in form of pairs of locations they want to have connected. Connections between locations are costly to build. The problem is to allocate costs of networks satisfying all connection demands. We use a few axioms to characterize allocation rules that truthfully implement cost minimizing networks satisfying all connection demands in a game where: (1) a central planner announces an allocation rule and a cost estimation rule; (2) every agent reports her own connection demand as well as all connection costs; (3) the central planner selects a cost minimizing network satisfying reported connection demands based on the *estimated* costs; and, (4) the planner allocates the *true* costs of the selected network. It turns out that an allocation rule satisfies the axioms if and only if relative cost shares are fixed.

JEL classification: C70; C72; D71; D85

Keywords: Axiomatic characterization; Connection networks; Cost sharing; Implementation; Truth-telling

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1. Introduction

Overview of the paper In the present paper we consider cost allocation in the connection network (CN) model used in Anshelevich et al. [1], Chen et al. [6], Juarez & Kumar [10] and Moulin [12]. There are finite sets of locations and agents. Agents have connection demands in form of pairs of locations they want to have directly or indirectly connected. Connections between locations are costly, undirected and congestion free. Consequently, several agents can use the same connection as part of paths satisfying their connection demands. Therefore connections are public goods. The CN model has broad empirical relevance as illustrated in Section 6 where we discuss the German Hansa as an example of network building and cost allocation in practice.

A minimum cost connection network (MCCN) is a network minimizing total cost subject to the constraint that all connection demands have to be satisfied. An allocation rule maps MCCNs, connection demands and connection costs to cost shares for all agents. Depending on the allocation rule there can be a potential conflict between overall welfare aimed at minimizing total cost and individual welfare aimed at minimizing individual cost shares. We characterize the set of allocation rules that truthfully implement MCCNs.

Two properties are at the heart of our characterization: *Unobserved Information Independence* (UII); and, *Network Independence* (NI). UII states that connection costs of unused connections should not influence cost shares. Two versions of UII are considered: UII for comparison of situations with the same set of locations and Strong UII (SUII) for comparison of situations with different sets of locations. NI states that cost shares should not depend on the selected MCCN in case of multiple MCCNs. In addition, *Scale Invariance* (SI) stating that cost shares are homogeneous of degree one in connection costs is considered. We characterize allocation rules satisfying UII or SUII and NI (Theorem 1 and Corollary 1) as well as UII or SUII, NI and SI (Corollaries 2 and 3).

The difference between UII and SUII is small. However, whether we use UII or SUII has some impact on the domain as well as the form of allocation rules. With UII we have to focus on connection networks with undemanded locations and allocation rules can depend on the sets of locations. With SUII we can consider all CNs and allocation rules cannot depend on the sets of locations.

In Theorem 1 and Corollary 1 we show that an allocation rule satisfies UII or SUII and NI if and only if cost shares depend on connection demands, total cost, the set of locations in case of UII, and nothing else. In Corollaries 2 and 3 we show that an allocation rule satisfies UII or SUII, NI and SI if and only if *relative* cost shares depend on connection demands, the set of locations in case of UII, and nothing else. Theorem 1 implies UII and NI are at odds with Individual Rationality where Individual Rationality means that no agent pays more than the cost of building a path satisfying her own connection demand.

In order to consider truthful implementation of MCCNs in the CN model we consider a variation of the game introduced in Hougaard & Tvede [8] where a planner is ignorant of connection demands and connection costs:

(1) A planner announces a cost allocation rule and a cost estimation rule.
(2) Every agent reports her connection demand and all connection costs.
(3) The planner selects a MCCN based on reported connection demands and *estimated* connection costs.
(4) The planner allocates the *true* costs of the selected network.