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Contagion and uninvadability in local interaction games: The bilingual game and general supermodular games ☆

Daisuke Oyama [a,*], Satoru Takahashi [b]

[a] *Faculty of Economics, University of Tokyo, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan*
[b] *Department of Economics, National University of Singapore, AS2 Level 6, 1 Arts Link, Singapore 117570, Singapore*

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Abstract

In a setting where an infinite population of players interact locally and repeatedly, we study the impacts of payoff structures and network structures on contagion of a convention beyond 2×2 coordination games. First, we consider the "bilingual game", where each player chooses one of two conventions or adopts both (i.e., chooses the "bilingual option") at an additional cost. For this game, we completely characterize when a convention spreads contagiously from a finite subset of players to the entire population in some network, and conversely, when a convention is never invaded by the other convention in any network. We show that the Pareto-dominant (risk-dominant, resp.) convention is contagious if the cost of bilingual option is low (high, resp.). Furthermore, if the cost is in a medium range, both conventions are each contagious in respective networks, and in particular, the Pareto-dominant convention is contagious only in some non-linear networks. Second, we consider general supermodular games, and compare networks in terms of their power of inducing contagion. We show that if there is a weight-preserving node identification from one network to another, then the latter is more contagion-inducing than the former in all supermodular games.

JEL classification: C72; C73; D83

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* Corresponding author.
E-mail addresses: oyama@e.u-tokyo.ac.jp (D. Oyama), ecsst@nus.edu.sg (S. Takahashi).
URLs: http://www.oyama.e.u-tokyo.ac.jp (D. Oyama), http://profile.nus.edu.sg/fass/ecsst (S. Takahashi).

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1. Introduction

Behavior initiated by a small group of individuals, such as adoption of languages or technology standards, can spread in the long run over a large population through local interactions. Such a phenomenon is called contagion (also known as diffusion or epidemics) and has attracted much attention (Blume [4,5], Ellison [8,9], Morris [33]).[1] In particular, focusing on the class of 2×2 coordination games, Morris [33] analyzes how contagion is affected by the payoffs of the game played as well as the topology of the underlying network. This paper considers the "bilingual game" and general supermodular games, thereby enriching our understanding of the impacts of payoff structures and network structures on strategic behavior in local interaction games.

To be specific, consider an infinite population of players who are connected with each other through a network. Suppose that each player uses one of two computer programming languages, or two types of technologies in general, A and B. The payoffs from each interaction with his neighbors are given by the following 2×2 coordination game:

	A	B
A	a, a	b, c
B	c, b	d, d

where $a > c$ and $d > b$, so that (A, A) and (B, B) are strict Nash equilibria.[2] We assume that $a > d$, i.e., (A, A) Pareto-dominates (B, B), while $a - c < d - b$, i.e., (B, B) risk-dominates (A, A). We further assume that $d \geq c$, so that coordination on some action is always better than miscoordination. Morris [33] shows that in 2×2 coordination games, the risk-dominant action B is always both contagious (i.e., in some network, there is some finite set of players such that if B is initially played by this set of players, then it is eventually played by the entire population) and uninvadable (i.e., in all networks, if B is initially played by almost all players, then it continues to be played by almost all players). Observe that B is a best response if at least a proportion $q = (a-c)/\{(a-c)+(d-b)\}$ of neighbors play B. Morris [33] defines the contagion threshold of a given network to be the supremum of the payoff parameter q such that contagion occurs in that network. In particular, the linear network with nearest neighbor interactions, as depicted in Fig. 1, has a contagion threshold $1/2$, which is the largest among all networks.

Now suppose that players can adopt a combination of the two actions, a "bilingual option" AB, with an additional cost $e > 0$. A player who plays AB receives a (gross) payoff a (d, resp.) from

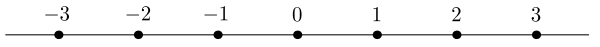

Fig. 1. Nearest neighbor linear interaction.

[1] See also Easley and Kleinberg [7], Goyal [14], Jackson [19], Vega-Redondo [44], Young [48], among others.

[2] With only two actions, the model is a special case of the "threshold model" (Granovetter [16]). For related studies in computer science, see, e.g., Easley and Kleinberg [7] or Wortman [47].