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A more general Pandora rule?

Wojciech Olszewski [a], Richard Weber [b,*]

[a] *Department of Economics, Northwestern University, Evanston, IL 60208, USA*
[b] *Statistical Laboratory, University of Cambridge, Cambridge CB3 0WB, UK*

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Abstract

In a model introduced by Weitzman an agent called Pandora opens boxes sequentially, in whatever order she likes, discovers prizes within, and optimally stops. Her aim is to maximize the expected value of the greatest discovered prize, minus the costs of opening the boxes. The solution, using the so-called Pandora rule, is attractive and has many applications. However, it does not address applications in which the payoff depends on all discovered prizes, rather than just the best of them, nor is it easy to say whether or not some generalized Pandora rule might do so. Here, we establish a sense in which it cannot. We discover that if a generalized Pandora rule is to be optimal for some more general utility, and all model parameters, then the problem can be solved via a second problem having Weitzman's form of utility.

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1. Introduction

In a classic problem first analyzed and solved by Weitzman (1979) an agent called Pandora is presented with n boxes, each of which contains a prize. Pandora can, by paying a known cost c_i, open box i to reveal its prize. The nonnegative value of the prize, denoted x_i^o, is not known until

* Corresponding author. Fax: +44 1223 337956.
E-mail address: rrw1@cam.ac.uk (R. Weber).

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the box is opened, but it has a distribution F_i that is known ex ante.[1] Pandora wishes to choose the order of opening the boxes, and when to stop opening, so as to maximize the expected value the greatest discovered prize net of the sum of the costs of opening boxes. Weitzman's problem is attractive for two reasons. Firstly, it has an enormous number of applications, such as to searching for a house, job, or research project to conduct.

Secondly, the solution is remarkably simple and attractive. Assign to any unopened box, say box i, a *reservation value* (or *reservation prize*), of

$$x_i^\dagger = \inf\{y : y \geq -c_i + E\max[y, x_i^o]\}, \tag{1}$$

the expectation being taken over x_i^o. This is the smallest prize value y for which the agent can do as well by taking y, as by opening box i and taking its prize or y. In (1) and throughout this note we are concerned only with Weitzman's problem in which costs and reward are undiscounted.

The so-called *Pandora rule*, which is optimal for Weitzman's problem, is: *open boxes in descending order of reservation values until a prize is found whose value weakly exceeds the reservation value of any unopened box.*

Attractive as it is, Weitzman's model does not cover an important and large class of problems in which the agent's utility is not only a function of the prize the agent takes when she stops, but of all the prizes uncovered. For example, a student may benefit from courses she takes while searching for the subject to choose as major; or a person may obtain a flow of utility by dating different partners while looking for a spouse; or an institution may be affected by different forms of operation with which it temporarily experiments before adopting a permanent one.

Weitzman expected that Pandora's rule would not generalize to such problems. He wrote: "*If some fraction of its reward can be collected from a research project before the sequential search procedure as a whole is terminated, that could negate Pandora's rule in extreme cases.*" However, Weitzman gave no supporting detailed analysis, and it turns out to be difficult to say whether or not some interesting generalization might be possible. In this note, we fill this gap in understanding by starting with a very general utility and propose a generalized Pandora rule. Then we take as "extreme cases" the requirement that this Pandora rule should be optimal for all choices of costs and prize value distributions, our motivation being that this is true in Weitzman's problem. We discover that if a generalized Pandora rule is to be optimal under this requirement then the problem can also be solved by solving a second problem having Weitzman's utility, using his Pandora rule.

2. Model

2.1. The generalized Pandora problem

An agent called Pandora is presented with n boxes, each of which contains a prize. Pandora can, by paying a known cost c_i, open box i to reveal its prize. The nonnegative value of the prize, denoted x_i^o, is not known until the box is opened, but it has a distribution function F_i which is known ex ante.

If S is the set of opened boxes at the point the agent stops, and the vector of the prize values found is $x_S^o = (x_i^o,\ i \in S)$, then the agent obtains a reward $u(x_S^o)$, expressed as a utility that

[1] The superscript 'o' is provided as a mnemonic for 'opened' or 'observed'.