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Complete and incomplete financial markets in multi-good economies

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Abstract

We investigate conditions for endogenous incompleteness and completeness in continuous-time financial markets driven by diffusion processes with multiple consumption goods and heterogeneous agents. We show that for a class of utility functions the financial market is endogenously incomplete. A sufficient condition for market completeness is that the dividend diffusion matrix in units of the numeraire good is invertible. Further, financial market completeness can depend on the choice of the numeraire good since changing the numeraire good implies a change of the risk-free asset and the asset structure.

JEL classification: G10; G11

Keywords: Multi-good economies; Financial market incompleteness; Financial market completeness

1. Introduction

We consider a continuous-time Lucas tree economy driven by diffusion processes with multiple consumption goods and agents with heterogeneous preferences. We derive sufficient conditions for financial market incompleteness and completeness without having to calculate the

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equilibrium stock price diffusion matrix. Instead, the conditions rely solely on the utility function of the representative agent and an invertibility condition on dividends.

First, we define a class of utility functions for which the span of the risky assets is strictly smaller than the span of the dividends. Hence, the financial market is incomplete. This class of utility functions covers the preferences employed, among others, in the following papers: Cole and Obstfeld (1991), Zapatero (1995), Serrat (2001), Cass and Pavlova (2004), and Berrada et al. (2007). Within this class is, for instance, the widely used Cobb–Douglas utility function. When the representative agent has Cobb–Douglas utility, as in Cole and Obstfeld (1991), then the commodity price is proportional to the relative dividends and, consequently, dividends measured in units of the numeraire correlate perfectly with each other. Hence, in equilibrium stock prices are linearly dependent.[1]

Second, we define a class of utility functions for which the financial market is complete. Specifically, our completeness condition only requires verification of an invertibility condition on dividends in terms of the numeraire good for one realization of the state variables at the terminal time. In contrast, to verify market completeness without such a condition one has to calculate the equilibrium stock price diffusion matrix and to check if the matrix is invertible for every possible realization of the state variables at every point in time. Without an explicit closed-form solution for the equilibrium stock price diffusion matrix, this is a hopeless task.

In a Lucas tree economy the typical asset structure consists of claims on the Lucas trees and a locally risk free asset in units of the numeraire good. We show that financial market completeness depends on the choice of the numeraire good, since changing the numeraire good and keeping the number of available assets fixed implies that the original risk-free asset is non-tradable under the new numeraire good. Therefore, changing the numeraire good can move an economy from incomplete to complete and vice versa. Numeraire good irrelevance holds[2] when our sufficient condition for incompleteness is satisfied; thus, for this class of utility functions the market remains incomplete under any numeraire good.

Even if the financial market is endogenously incomplete, the numeraire good can be important. We show that the choice of numeraire good can determine whether trading in the available assets implements the Arrow–Debreu equilibrium. For example, for a certain choice of the numeraire good the equilibrium is of the peculiar type as in Cass and Pavlova (2004), even though agents do not have log-linear utility functions. For any other choice of the numeraire good, the endogenous incompleteness has real effects, as agents cannot implement the Arrow–Debreu equilibrium.

The departure point for our work is that in several multi-good models in the literature the span of the stocks drops relative to the span of the dividends. Specifically, Serrat (2001) solves a continuous-time international (Lucas, 1978) tree economy with multiple consumption goods and derives an explicit formula for stock price diffusion coefficients. Yet, it appears that even with an explicit formula for stock price diffusion coefficients it can be difficult to detect an inherently incomplete market. In the end, Kollmann (2006) shows that the economy studied by Serrat (2001) has incomplete financial markets. Importantly, Serrat (2001) claims that the presence of non-

[1] Rosenberg and Ohlson (1976) study a related problem in multi-asset purely financial models with fixed asset supplies. Assuming that the aggregate investor has constant relative risk aversion utility and that asset prices follow a joint lognormal process implies that asset proportions are constant. In equilibrium, this can only be the case if risky asset returns are perfectly correlated.

[2] We use the term *numeraire good irrelevance* to mean that the financial market does not switch from complete to incomplete when we change the numeraire good.