Aggregation theory and the relevance of some issues to others

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Abstract

I propose a relevance-based independence axiom on how to aggregate individual yes/no judgments on given propositions into collective judgments: the collective judgment on a proposition depends only on people's judgments on propositions which are *relevant* to that proposition. This axiom contrasts with the classical independence axiom: the collective judgment on a proposition depends only on people's judgments on *the same* proposition. I generalize the premise-based rule and the sequential-priority rule to an arbitrary priority order of the propositions, instead of a *dichotomous* premise/conclusion order resp. a *linear* priority order. I prove four impossibility theorems on relevance-based aggregation. One theorem simultaneously generalizes Arrow's Theorem (in its general and indifference-free versions) and the well-known Arrow-like theorem in judgment aggregation.
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1. Introduction

The judgment aggregation problem consists in merging many individuals' judgments ('yes' or 'no') on some interconnected propositions into collective judgments on these propositions. Judgment aggregation ('JA') has wide applications. A classic example is decision-making in a jury in court, where the jurors have to merge their judgments on three controversial propositions: (i) the defendant has broken the contract; (ii) the contract is legally valid; (iii) the defendant is guilty (e.g., Kornhauser and Sager, 1986; List and Pettit, 2002). These propositions are interconnected because legal doctrine prescribes that (iii) holds if and only if (i) and (ii) both hold. Another example is *preference aggregation*. Here we merge people's judgments on propositions of the kind 'option x is weakly preferable to option y' – in short: $x R y$ – for various pairs of options x and y (where these propositions are interconnected via conditions such as transitivity). In yet another example, we merge people's estimates of some variables (such as GDP, prices and unemployment). In other words, we merge people's judgments on propositions of the sort 'variable k takes value v' for various pairs of a variable k and a possible value v (where these propositions might be interconnected via some macroeconomic equations). Similarly, we might merge grades which people give to some politicians, where the possible grades might be 'good', 'excellent' and 'bad' (as in Balinski and Laraki's 2010 voting theory). In other words, we merge people's judgments on propositions of the sort 'politician k is of quality v' for pairs of a politician k and a possible grade v. The last two examples are versions of the *evaluation aggregation* problem, in which we merge people's positions on some matters: people's estimates of variables, people's grades given to politicians, people's degrees of belief in some events, etc. (e.g., Rubinstein and Fishburn, 1986; Dietrich and List, 2010; Dokow and Holzman, 2010b).

Evidently, many 'special' aggregation problems can be stated as JA problems – but does JA theory have to say something interesting about them? JA theory has been particularly successful at generalizing theorems and insights from *preference* aggregation theory, including Arrow's Theorem in its indifference-free version. JA theory has been less successful at addressing some other aggregation problems, including preference aggregation in its general (indifference-permitting) form, the aggregation of (non-binary) evaluations, and the aggregation of judgments on propositions with a more complex priority structure than a dichotomous premise/conclusion structure. Perhaps the main reason is that JA theory draws strongly on the classic but controversial axiom of *proposition-wise independence*: the collective judgment on a proposition should be determined solely by people's judgments on *this* proposition. This axiom denies that other propositions can be relevant. I call a proposition p 'relevant' to another q if people's judgments on p matter for forming the collective judgment on q, so that the latter should draw on the former. Proposition-wise independence implicitly assumes a narrow notion of relevance: each proposition is relevant only to itself. The implausibility of the axiom and its narrow relevance notion becomes evident in our introductory examples:

(a) In the jury example, the popular *premise-based* procedure violates proposition-wise independence and treats the two 'premise propositions' (i) and (ii) as relevant to the 'conclusion proposition' (iii), since the collective judgment on (iii) is derived from jurors' judgments on (i) and (ii). (More precisely, the collective endorses (iii) if and only if each premise proposition is endorsed by majority.) There are many other examples of propositions between which there are relevance connections of a premise-conclusion type, making proposition-wise independence implausible.