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Rational behavior under correlated uncertainty ☆

Mira Frick [a,*], Assaf Romm [b,*]

[a] *Cowles Foundation for Research in Economics, Yale University, United States*
[b] *Department of Economics, Hebrew University, Israel*

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Abstract

In complete information games, Dekel and Fudenberg (1990) and Börgers (1994) have proposed the solution concept $S^{\infty}W$ (one round of elimination of weakly dominated strategies followed by iterated elimination of strongly dominated strategies), motivating it by a characterization in terms of "approximate common certainty" of admissibility. We examine the validity of this characterization of $S^{\infty}W$ in an *incomplete* information setting. We argue that in Bayesian games with a nontrivial state space, the characterization is very sensitive to the way in which uncertainty in the form of approximate common certainty of admissibility is taken to interact with the uncertainty already captured by players' beliefs about the states of nature: We show that $S^{\infty}W$ corresponds to approximate common certainty of admissibility when this is not allowed to coincide with any changes to players' beliefs about states. If approximate common certainty of admissibility is accompanied by vanishingly small perturbations to beliefs, then $S^{\infty}W$ is a (generally strict) subset of the predicted behavior, which we characterize in terms of a generalization of Hu's (2007) perfect p-rationalizable set.

JEL classification: C70; C72

Keywords: Rationality; Admissibility; Approximate common certainty; Common p-belief; Incomplete information; Robustness

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* Corresponding authors.
E-mail addresses: mira.frick@gmail.com (M. Frick), assafr@gmail.com (A. Romm).

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1. Introduction

In complete information games, Dekel and Fudenberg (1990) and Börgers (1994) have proposed the solution concept $S^\infty W$ (one round of elimination of weakly dominated strategies followed by iterated elimination of strongly dominated strategies), motivating it via its connection with "approximate common certainty" of admissibility. Admissibility (expected utility maximization with respect to some full-support conjecture about opponents' behavior) and iterated admissibility are commonly used refinements of Bayesian rationality (e.g. Luce and Raiffa, 1957; Kohlberg and Mertens, 1986).

Börgers's interest in approximate common certainty of admissibility is driven by epistemic considerations, namely the aim to establish an analog of Tan and da Costa Werlang's (1988) well-known result that the behavioral implications of common certainty of rationality are given by S^∞ (iterated strong dominance). There is a logical tension between admissibility (holding full-support beliefs about opponents' behavior) and common certainty of admissibility (which in general rules out some opponent strategies). This tension disappears when common certainty is relaxed to approximate common certainty in the sense of common p-belief for p close to 1. Börgers (formalized by Hu, 2007) shows that $S^\infty W$ encapsulates the behavioral implications of the latter notion. Dekel and Fudenberg are motivated by robustness considerations, focusing on the special case where approximate common certainty of admissibility is the result of small amounts of payoff uncertainty. They ask which strategies can arise if players behave according to iterated admissibility, but there is vanishingly small payoff uncertainty, which they model via sequences of elaborations converging to a game. Once again the answer to this question is given by $S^\infty W$.

This paper examines the connection between approximate common certainty of admissibility and $S^\infty W$ in an *incomplete* information setting. Consider a Bayesian game G with state space Θ in which each player i has first-order belief ϕ_i over Θ. We obtain extensions of Börgers's and Dekel and Fudenberg's characterizations of $S^\infty W$, but show that these are very sensitive to the way in which uncertainty in the form of approximate common certainty of admissibility is taken to interact with the uncertainty (represented by each player i's belief ϕ_i on Θ) that is already present in G.

Interpreting $S^\infty W$ in the interim-correlated sense of Dekel et al. (2007), Section 3.1 extends Börgers's characterization: We show that if there is common p-belief of admissibility *and* of the fact that each player i's first-order belief over Θ is *exactly* ϕ_i, then for p close enough to 1, $S^\infty W$ once again emerges as the set of behavioral implications (Theorem 3.1). In Appendix A, we provide an analogous extension of Dekel and Fudenberg's result: Proposition A.2 shows that $S^\infty W$ is the robust extension of W^∞ under elaborations in which "sane" types may assign vanishingly small probability to opponents being "crazy" (i.e. having very different payoffs and beliefs as in the original game), but must themselves have *exactly* the same beliefs (and payoffs) as in the original game.

However, these results break down when approximate common certainty of admissibility is accompanied by vanishingly small perturbations to players' beliefs about states. In Section 3.2 we show that if there is common p-belief of admissibility and of the fact that each player i's first-order belief about Θ is "approximately" ϕ_i, then the behavioral implications are given by a generalization of Hu's (2007) perfect p-rationalizable set (Theorem 3.5). But even in the limit