Reputation in the long-run with imperfect monitoring ☆

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Abstract

We study an infinitely repeated game where two players with equal discount factors play a simultaneous-move stage game. Player one monitors the stage-game actions of player two imperfectly, while player two monitors the pure stage-game actions of player one perfectly. Player one's type is private information and he may be a "commitment type," drawn from a countable set of commitment types, who is locked into playing a particular strategy. Under a full-support assumption on the monitoring structure, we prove a reputation result for stage games with a strong Stackelberg action: if there is positive probability that player one is a particular type whose commitment payoff is equal to player one's highest payoff, consistent with the players' individual rationality, then a patient player one secures this type's commitment payoff in any Bayes–Nash equilibrium of the repeated game. In contrast, if the type's commitment payoff is strictly less than player one's highest payoff consistent with the players' individual rationality, then the worst perfect Bayesian equilibrium payoff for a patient player one is equal to his minimax payoff.
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1. Introduction

A patient player's reputation concerns are the dominant incentives that determine equilibrium payoffs in repeated games where a patient player faces a myopic opponent. And this is true regardless of the monitoring structure.[1] Building a reputation when facing an equally patient opponent, however, is more difficult. A patient opponent might be willing to sacrifice short-term payoffs to test whether the player, who is trying to build a reputation, will go through with his threats or promises. This makes it prohibitively expensive to build a reputation in certain repeated simultaneous-move games played against a patient opponent if stage-game actions are *perfectly monitored* (Cripps and Thomas [11]). In this paper, we instead focus on repeated simultaneous-move games played by equally patient players where the opponent's stage-game actions are *imperfectly monitored*. We show that reputation effects are prominent under imperfect monitoring even in certain repeated games where reputation effects are absent under perfect monitoring.

Specifically, suppose that player one's type is private information and that he may be one of many finite-automaton "commitment types" each of whom is locked into playing a particular repeated-game strategy. We explore whether an uncommitted or "normal" player can exploit his opponent's uncertainty to establish a reputation and thereby guarantee an advantageous equilibrium payoff. Our central finding is a lower bound on player one's (he) equilibrium payoff for repeated games where he observes only an imperfect public signal of his opponent's stage-game action while his opponent perfectly monitors player one's actions. This lower bound is tight if the set of commitment types includes a pure-strategy finite-automaton commitment type with *no shortfall*, i.e., a type with a commitment payoff that is equal to player one's highest payoff compatible with the players' individual rationality (player one's highest IR payoff).[2] In particular, if a finite automaton with no shortfall is available, then a patient player one can guarantee his highest IR payoff in any Bayesian–Nash equilibrium of the repeated game. Player one can guarantee his highest IR payoff by simply mimicking the strategy of the finite automaton with no shortfall, even if player two believes that player one is another finite automaton with arbitrarily higher probability.

The lower bound that we establish implies a reputation result for certain repeated games where there exists a pure stage-game action (*a strong Stackelberg action*) which has a commitment payoff equal to player one's highest IR payoff.[3] In a repeated game where there is a strong Stackelberg action, the commitment type which plays the strong Stackelberg action in each period of the repeated game is a pure strategy finite automaton with no shortfall. Therefore, in such a repeated game, player one can guarantee his highest IR payoff by simply mimicking this type, that is by playing the strong Stackelberg action in each period of the repeated game.

We turn next to the question of whether player one can still benefit from a reputation even if the shortfall for the only available commitment type is positive (i.e., the type's commitment payoff is less than player one's highest IR payoff). In this case, we show that a patient player one's worst equilibrium payoff is equal to his minimax. Therefore, player one guarantees only his lowest

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[1] See Fudenberg and Levine [16] for the case of perfect monitoring, Fudenberg and Levine [17] for imperfect public monitoring, and Gossner [19] for imperfect private monitoring.

[2] The commitment payoff of a type is the payoff that player one can guarantee by publicly committing to play the repeated-game strategy that this type plays. A type's (or strategy's) *shortfall* is the difference between player one's highest IR payoff and the type's commitment payoff as the discount factor goes to one.

[3] The Stackelberg payoff for player one is the highest payoff he can guarantee in the stage-game through public commitment to a stage-game action (a Stackelberg action). See Mailath and Samuelson [20, page 465], for a formal definition.