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Decreasing aversion under ambiguity

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Abstract

Under which condition does the set of desirable uncertain prospects expand when wealth increases? We show that the decreasing concavity (DC) of the utility function u is necessary and sufficient in the α-maxmin expected utility model. In the smooth ambiguity aversion model with the ambiguity valuation function ϕ, the DC of u and of $\phi \circ u$ is necessary and sufficient. An alternative classical definition of decreasing aversion is based on the hypothesis that the investment in a risky asset is increasing in wealth. We show that this hypothesis does not hold in general under ambiguity aversion, and that one needs to constrain the structure of ambiguity to obtain unambiguous results.

JEL classification: D81

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1. Introduction

One of the most ubiquitious assumption in the economics of risk is that wealthier people are less risk-averse. Various definitions of the concept of decreasing aversion exist in the literature. For example, an agent is said to have decreasing aversion if any risk that is undesirable

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at some specific wealth level is also undesirable at any smaller wealth level. Another definition of decreasing aversion is that in the one-risk-free-one-risky-asset portfolio choice problem, the demand for the risky asset is an increasing function of initial wealth. In the classical expected utility model, these two definitions of decreasing aversion are equivalent, and the necessary and sufficient condition is expressed by the decreasing nature of the Arrow–Pratt index of absolute risk aversion (DARA).

Since Arrow [2] postulated decreasing absolute risk aversion, numerous studies have confirmed this hypothesis using both experimental (Levy [22], Guiso and Paiella [18]) and econometric (Bar-Shira et al. [3]) methods. This universally accepted property of individual risk preferences plays a crucial role in many applications of the expected utility theory, as illustrated in Gollier [14]. Risk aversion is indeed essential to understand individual choices about wealth accumulation, retirement, portfolio accumulation and insurance. Somehow, how risk aversion shapes economic decision is still an open issue. In particular, the observed level and share[2] of savings invested in risky assets is not well predicted by existing models. Recent works enrich these models by taking into account labor income (e.g. Viceira [29], Cocco et al. [8]) or housing (e.g. Cocco [7], Yao and Zhang [30]), or by refining preferences with persistent habits (Gomes and Michaelides [16], Brunnermeier and Nagel [5]) and more recently ambiguity aversion (Campanale [6]).

In this paper, we explore the concept of decreasing aversion in the context of ambiguity and ambiguity aversion. In most cases, the probability distribution of the risk is not perfectly known, i.e., it is ambiguous. Examining a simple thought experiment, Ellsberg [10] suggested that economic agents do not behave accordingly to the subjective expected utility model. Under ambiguity, contrary to Savage [27] theory of subjective expected utility, they do not use a subjectively chosen probability distribution to compute the expected utility of the set of possible acts to determine their optimal strategy. Many experiments have confirmed Ellsberg's hypothesis that in the absence of an objective probability distribution, individuals tend to favour a relatively pessimistic plausible distribution to measure their welfare ex ante. Gilboa and Schmeidler [11] were the first to propose a decision criterion (thereafter refered to as maxmin) that is compatible with Ellsberg's hypothesis, and that generalizes the expected utility model. In short, agents are assumed to have multiple priors whose formation is a characteristic of the preferences of the agent. The agent's ex ante welfare associated to an act is the smallest expected utility generated by this act over the different possible priors.

More recently, two models have been proposed to account for ambiguity attitude. Ghirardato, Maccheroni and Marinacci [12] have proposed the α-maxmin expected utility (or α-MEU) family of preferences in which the agent's ex ante welfare is measured by an α-weighted average of the smallest and the largest expected utility levels among a convex, compact set of probability distributions. The alternative approach (thereafter refered to as KMM) provided by Klibanoff, Marinacci and Mukerji [21] represents the agent's welfare under uncertainty by the certainty equivalent of the different prior-dependent expected utility levels. This certainty equivalent is

[2] Another property central to this literature is relative risk aversion (RRA), that determines how the *share* invested in the risky asset evolves with initial wealth in the one-risk-free-one-risky-asset portfolio choice problem. Here, we focus on absolute rather than relative risk aversion, since the DARA property has been validated by most empirical studies whereas empirical evidence regarding RRA are not conclusive: some empirical studies support decreasing RRA (Cohn et al. [9], Morin and Suarez [23], Levy [22], Guiso et al. [17], Ogaki and Zhang [24]), other studies are in favor of increasing RRA (Siegel and Hoban [28], Bar-Shira et al. [3], Barksy et al. [4]). It must be noted that decreasing relative risk aversion is sufficient but not necessary to decreasing absolute risk aversion.