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Integrated digital image correlation for the evaluation and correction of optical distortions

John-Eric Dufour, François Hild*, Stéphane Roux

Laboratoire de Mécanique et Technologie (LMT-Cachan), ENS Cachan/CNRS/PRES UniverSud Paris, 61 Avenue du Président Wilson, F-94235 Cachan Cedex, France

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ABSTRACT

Optical aberrations are one of the biases affecting images. Their correction can be performed as a transformation of the image. In this paper, it is proposed to use digital image correlation to calculate the distortion fields. A correlation calculation is performed between a known numerical pattern and a picture of the same pattern printed or etched on a plate. This new procedure is applied to determine the distortion fields of a camera lens. First, a parametric description of the distortion field is used to reduce the degrees of freedom required to describe optical distortions. Second, a non-parametric model based upon splines is used. The distortion fields found by both methods are compared. A resolution analysis is performed for two different procedures (removing the lens between pictures or not). The standard displacement resolution is found to be of the order of 10^{-2} pixel in the first case, and 2.5×10^{-3} pixel in the second case.

1. Introduction

Digital image correlation consists of the registration of pictures taken by cameras. The mechanism of taking a picture with a digital camera induces some differences between the ideal geometrical projection of the real scene and the acquired picture. As the definition of the camera sensors increases, the main source of errors is now due to lenses generating geometrical aberrations resulting in a distortion field representing the displacement field between the real scene and the shot one. Digital image correlation uses pictures to measure displacements, therefore these systematic errors bias the measurements [1–7]. That is the reason why a correction of these distortions has to be performed so that measurements become quantitative and bias-free.

Several procedures have already been proposed to measure and correct distortions so as to restore the geometrical fidelity of images. Many of these methods use geometrical calibration targets (*e.g.*, grids and checkered surfaces) to perform the correction. The latter consists of linear coordinate transformations for non-parametric[1] descriptions with no relationship to the physics of distortion phenomena. Other techniques use similar methods to obtain stereocorrelation calibration matrices and distortion correction procedures [8,9,11,4,5]. This allows a single calibration step to achieve both objectives. Another strategy consists of using a parametric description of the distortion field. The latter is extracted and then fitted to obtain a set of parameters [2,1,12]. This method generally uses grids to obtain the distortion field but implies a modification of the image, which is a non-linear transformation (addition of a new field to the image). Some approaches use both non-parametric model and nonlinear transformation. For instance, a spline-based description [12] can be used to correct for distortions. With these methods, only a few reference points are extracted from the whole pictures of the calibration target. The most used registration techniques are based on geometrical elements (for example circles or intersecting lines) to determine remarkable points on a target using *e.g.* Harris' corner detector [13]. Correlation-based approaches are another alternative [12,5] used herein. In contrast with the previous method, a dense measurement of an apparent displacement field is performed herein with a uniform measurement over the region of interest, and it does not rely on interpolation or regression procedures in between remarkable points.

It is proposed to use an integrated approach to DIC directly considering a parametric description of the distortion field. Although most methods are based upon regular geometrical targets, the

* Corresponding author. Tel.: +33 1 47 40 21 92; fax: +33 1 47 40 22 40.
E-mail addresses: john-eric.dufour@lmt.ens-cachan.fr (J.-E. Dufour), francois.hild@lmt.ens-cachan.fr (F. Hild), stephane.roux@lmt.ens-cachan.fr (S. Roux).

[1] The terminology "parametric" refers to a model-based representation of the measurement so that the parameters can be directly interpreted as the characteristics of a physical model. In contrast a discretization based on a mathematically convenient basis without direct physical interpretation will be called "non-parametric" to comply with a common convention [8–11,5].

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present analysis is based on a printed or etched random pattern to extract the distortions from the whole picture. The integrated approach allows us to directly extract the parameters from the DIC analysis without any additional interpolation. It can be used in addition to a spline basis to calculate the parameters of a spline-based correction of distortion.

The outline of the paper is as follows. First, the concept of optical distortion is introduced by defining the underlying physical biases. Then the method to evaluate distortions is presented. It is based on an integrated DIC approach. A resolution analysis is also performed by both analytical and experimental approaches. Last, a spline-based method and its resolution analysis are compared for two different basis orders.

2. Optical distortions

In the sequel, the main optical distortions and the mathematical model to account for these biases are introduced.

2.1. Physics of distortions

Optical distortions are mainly caused by imperfections of cameras and their optics. Different kinds of defaults generate optical distortions. Thickness of the lenses and alignments in the optical system are generally approximations of a camera model and are not perfect. Several types of optical biases are generally identified [2,8,5] and only three of them are listed and studied:

- Radial distortions are due to the paraxial approximation. When light comes into the system with a large angle from the optical axis, the Gaussian approximation of its path does not represent the reality. These rays do not hit the CCD (or CMOS) sensor at the location predicted by the model, thereby causing a distortion [1].
- Decentering distortions are caused by the lack of coaxiality of the center of the lenses composing the optics. In an aligned optical system, rays traveling along the center of the lenses are not deviated. If the different centers are not coaxial, all rays deviate. Although an 'apparent' optical axis can be defined to take into account the whole system, it does not coincide with the optical center predicted by the perfect camera model [1].
- Thin prism distortions are due to the fact that the parallelism of the lenses between each other and with the CCD or CMOS sensor is not completely satisfied. This last distortion is also known as tilt distortion [1].

Optical biases can be described as an apparent displacement field between an image taken with a real camera and a perfect image of the same scene.

2.2. Apparent motions

For each distortion described above, a mathematical model exists. It accounts for the fact that the true pixel location (x_r, y_r) is biased and therefore has 'moved' by an apparent displacement (δ_x, δ_y) so that its position in the picture is $(x_r+\delta_x, y_r+\delta_y)$. Correcting optical distortions consist of determining the apparent motions (δ_x, δ_y) [1,2]. To correct for distortions, the distorted image has to be corrected to bring it back to the reference (*i.e.*, undistorted) configuration. This is precisely what is performed in the registration operation in any DIC code (see below). The only difference is that the displacement field $(\delta_x(x_r, y_r), \delta_y(x_r, y_r))$ is known since the distortion components have been determined, as shown in the sequel. Consequently, correcting for distortions consists of calculating $\tilde{g}(x_r, y_r) = g(x_r+\delta_x(x_r, y_r), y_r+\delta_y(x_r, y_r))$ for any considered pixel location (x_r, y_r). Since g is known at the pixel locations (x_r, y_r), $g(x_r+\delta_x(x_r, y_r), y_r+\delta_y(x_r, y_r))$ will require evaluations at noninteger positions. Thus, gray level interpolations are needed, which are identical to those used in the DIC procedure.

Radial distortions: The radial distortions are related to the series of odd powers of the radial coordinate of a pixel. They are described as an infinite series [1,2]

$$\delta_x^R(x_r, y_r) = x \sum_{k=1}^{\infty} r_k (x_r^2+y_r^2)^k$$
$$\delta_y^R(x_r, y_r) = y \sum_{k=1}^{\infty} r_k (x_r^2+y_r^2)^k \quad (1)$$

where δ_x^R, δ_y^R denote the apparent displacements due to radial distortions, and r_k are the corresponding (unknown) amplitudes.

Decentering distortions: This model represents the influence of having an apparent optical center, which is different from the assumed one. It is also expressed as an infinite series [1,2]

$$\delta_x^D(x_r, y_r) = (2x_r y_r \cos\phi_0 - (3x_r^2+y_r^2)\sin\phi_0) \sum_{k=1}^{\infty} s_k (x_r^2+y_r^2)^{k-1}$$
$$\delta_y^D(x_r, y_r) = (2x_r y_r \sin\phi_0 - (x_r^2+3y_r^2)\cos\phi_0) \sum_{k=1}^{\infty} s_k (x_r^2+y_r^2)^{k-1} \quad (2)$$

where δ_x^D, δ_y^D denote the apparent displacements due to decentering distortions, s_k the corresponding (unknown) amplitudes, and ϕ_0 the angular position of the apparent center in the picture.

Thin prism distortions: This bias is related to the series of even powers of the radial coordinate. An infinite series [2,1] is also utilized

$$\delta_x^P(x_r, y_r) = -\sin\phi_0 \sum_{k=1}^{\infty} t_k (x_r^2+y_r^2)^k$$
$$\delta_y^P(x_r, y_r) = \cos\phi_0 \sum_{k=1}^{\infty} t_k (x_r^2+y_r^2)^k \quad (3)$$

where δ_x^P, δ_y^P are the apparent displacements due to prismatic distortions and t_k the corresponding (unknown) amplitudes.

In this study, we will use a first order expansion ($k=1$) of the previous expressions:

$$\delta_x^R(x_r, y_r) = r_1 x_r (x_r^2+y_r^2)$$
$$\delta_y^R(x_r, y_r) = r_1 y_r (x_r^2+y_r^2)$$
$$\delta_x^S(x_r, y_r) = 2d_2 x_r y_r - d_1(3x_r^2+y_r^2)$$
$$\delta_y^S(x_r, y_r) = 2d_1 x_r y_r - d_2(3x_r^2+y_r^2)$$
$$\delta_x^P(x_r, y_r) = p_1(x_r^2+y_r^2)$$
$$\delta_y^P(x_r, y_r) = p_2(x_r^2+y_r^2) \quad (4)$$

This restriction to the first orders allows us to keep a few unknowns for the system (*i.e.*, 5 coefficients) while modeling the most salient features. This is the standard hypothesis made to model distortions [1,2,8,5].

3. Distortion evaluation via integrated-DIC

In this section, Digital Image Correlation [3,5,14] will be used. Contrary to the common practice of DIC, which is local (*i.e.*, subset-based), the whole ROI will be analyzed as a single interrogation window. Two different approaches will be considered. First, when a non-parametric distortion model is assumed, a spline-based displacement basis is chosen. This type of approach was already used to analyze beam bending [15] and wood sample compression [16]. An alternative is to resort to so-called integrated-DIC [17,18] in which the sought displacement field is directly given by, say, Eq. (4). There is therefore no need to re-project the measured displacement field.

To perform DIC analyses, several images are required. Let us first consider an image generated numerically as a random pattern of black dots on a white field. This picture is considered as the