Available online at www.sciencedirect.com
ScienceDirect
Journal of Economic Theory 157 (2015) 730–762
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Put–Call Parity and market frictions ☆

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Received 6 November 2013; final version received 5 September 2014; accepted 16 December 2014
Available online 6 January 2015

Abstract

We extend the Fundamental Theorem of Finance and the Pricing Rule Representation Theorem to the case in which market frictions are taken into account but the Put–Call Parity is still assumed to hold. In turn, we obtain a representation of the pricing rule as a discounted expectation with respect to a *nonadditive* risk neutral probability.

JEL classification: G12; G13; D81

Keywords: Put–Call Parity; Market frictions; Fundamental Theorem of Finance; No arbitrage; Choquet pricing

1. Introduction

We extend the Fundamental Theorem of Finance and the Pricing Rule Representation Theorem to markets with frictions.[1] We assume the Put–Call Parity and the absence of arbitrage opportunities and, under these hypotheses, we obtain a representation of the pricing rule as a discounted expectation with respect to a *nonadditive* risk neutral probability. In other words,

☆ We thank Alain Chateauneuf and Paolo Guasoni for very useful discussions and comments. Simone Cerreia-Vioglio and Fabio Maccheroni gratefully acknowledge the financial support of MIUR (PRIN grant 20103S5RN3_005) and of the AXA Research Fund. Massimo Marinacci gratefully acknowledges the financial support of the European Research Council (advanced grant BRSCDP-TEA) and of the AXA Research Fund.
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[1] The combination of these two results is also known in the literature as the Fundamental Theorem of Asset Pricing (see, e.g., [16,12]).

http://dx.doi.org/10.1016/j.jet.2014.12.011
0022-0531/

the market prices contingent claims as an ambiguity sensitive, but risk neutral, decision maker. As a further contribution, we remove the state space structure and the contingent claim representation that are usually assumed exogenously to model assets and markets. In particular, this allows us to provide a unique mathematical framework where we can both discuss the Fundamental Theorem of Finance and the Pricing Rule Representation Theorem.

Most of the fundamental theory of asset pricing relies on two main hypotheses: frictionless markets and absence of arbitrage.[2] On the other hand, frictions and transaction costs are present in financial markets and play an important role. Important evidence of these facts is the existence of bid–ask spreads (see, e.g., Amihud and Mendelson [3,4]). As a consequence, the Finance literature developed models that incorporate transaction costs and taxes (see, e.g., Garman and Ohlson [17], Prisman [35], Ross [39], Jouini and Kallal [26], and Luttmer [32]). In particular, [35,26,32] observe how taxes/transaction costs generate pricing rules that are not linear but still can be compatible with the no arbitrage assumption. Inter alia, Prisman [35] shows that convex transaction costs or taxes generate convex pricing rules. Furthermore, if transaction costs are different among securities but proportional to the volumes dealt, then the respective pricing rules are sublinear, as in [26,32].

Our approach is different. In a standard framework, the no friction assumption paired with the Law of One Price yields the fundamental Put–Call Parity, first discovered by Stoll [44] (see also Kruizenga [30]). Moreover, the no friction assumption also implies that when a risk-free position is added to an existing portfolio the price of the resulting portfolio is equal to the price of the original portfolio plus the price of the position on the risk-free asset. This last implication is basically equivalent to say that the price on the market of the risk-free asset is linear and in particular the bid–ask spread is zero on this market. From an applied point of view, the absence of frictions on the market of the risk-free asset and the Put–Call Parity are two important assumptions since they can be empirically tested.[3] These two joint properties are at the center of our study.

We study price functionals and pricing rules that satisfy a version of the Put–Call Parity and exhibit no frictions in the market of the risk-free asset. These two no frictions assumptions are conceptually much weaker than the standard one and much easier to test empirically. As in the standard case, we further retain a no arbitrage postulate. We show (Theorems 1 and 3) that these pricing rules can be characterized as discounted expectations with respect to a *nonadditive* probability, that is, by using Choquet expectations. One important feature of our result is that Choquet pricing rules are characterized by preserving the aforementioned financial identities but, a priori, they are not the direct result of assuming transaction costs, bid–ask spreads, or short-sales constraints. Instead, making these assumptions would naturally lead to sublinear pricing rules which have been studied exactly to account for transaction costs (as in Jouini and Kallal [26] and Luttmer [32]; see also Kabanov and Safarian [27]). It is then natural to ask what is the overlap between Choquet and sublinear pricing rules. Corollaries 1 and 2 show that a pricing rule is sublinear and Choquet if and only if the nonadditive probability that represents it is concave. In this case, the set of consistent price systems coincides with the core of this nonadditive probability. Thus, among the others, we provide testable conditions under which transaction costs generate a sublinear pricing rule which is also a nonadditive expectation.

[2] See, e.g., Ross [36,38], Cox and Ross [9], and, in a dynamic setting, Harrison and Kreps [23], Harrison and Pliska [24], and Delbaen and Schachermayer [11]. For an introduction to the topic, see Dybvig and Ross [14], Ross [40], Follmer and Schied [16], and Delbaen and Schachermayer [12].

[3] The empirical validity of the Put–Call Parity condition should be tested by using European options data, like in Kamara and Miller [28], in order to avoid issues of early exercise.