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Exchangeable capacities, parameters and incomplete theories ☆

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Abstract

The de Finetti Theorem on exchangeable predictive priors is generalized to a framework where preference is represented by Choquet expected utility with respect to a belief function (a special capacity). The resulting model provides behavioral foundations for the decision-maker's subjective theory of the environment in which there are factors common to all experiments (or sources of uncertainty), called parameters, but in which her theory is incomplete in that knowledge of the parameter leaves idiosyncratic factors that vary across experiments in a way that is poorly understood.

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1. Introduction

1.1. Outline

Let a family of experiments be indexed by the set $\mathbb{N} = \{1, 2, \ldots\}$. Each experiment yields an outcome in the set S (technical details are suppressed in this section). Thus $\Omega = S^\infty$ is the set of all possible sample paths. A probability measure P on Ω is *exchangeable* if

$$P(A_1 \times A_2 \times \ldots) = P(A_{\pi^{-1}(1)} \times A_{\pi^{-1}(2)} \times \ldots),$$

for all finite permutations π of $\mathbb{N}$. De Finetti [16] shows that exchangeability is equivalent to the following representation: There exists a (necessarily unique) probability measure μ on $\Delta(S)$ such that

$$P(\cdot) = \int_{\Delta(S)} \ell^\infty(\cdot)\, d\mu(\ell), \tag{1.1}$$

where, for any probability measure ℓ on S (written $\ell \in \Delta(S)$), ℓ^∞ denotes the corresponding i.i.d. product measure on Ω. Thus beliefs are i.i.d. conditional on the unknown parameter ℓ; learning is then modeled by Bayesian updating of beliefs about the parameter. Kreps [26, Ch. 11] refers to de Finetti's celebrated result as "the fundamental theorem of (most) statistics" because of the justification it provides for the analyst to view samples as being independent and identically distributed with unknown distribution function; and he argues for the importance of exchangeability and de Finetti's Theorem as normative decision tools.

Though the de Finetti Theorem can be viewed as a result in probability theory alone, it is typically understood in economics as a prescription for imposing structure on the predictive prior P in the subjective expected utility model of choice. That is also how we view it and accordingly we provide a decision-theoretic generalization of de Finetti's result that we view as largely normative. Specifically, we consider preference on a domain of (Anscombe–Aumann) acts that conforms to Schmeidler's [35] Choquet expected utility where the capacity is a belief function–we call this model *belief function utility*.[1] Using the latter as the basic framework, we then impose two simple axioms–Exchangeability (the preference counterpart of de Finetti's assumption) and Weak Orthogonal Independence (relaxing the Independence axiom). These axioms are shown (Theorem 3.1) to characterize the following representation for the belief function κ on Ω (see the noted theorem for the corresponding representation of utility):

$$\kappa(\cdot) = \int_{Bel(S)} \nu^\infty(\cdot)\, d\mu(\nu), \tag{1.2}$$

where $Bel(S)$ denotes the set of all belief functions on S, μ is a (necessarily unique) probability measure on $Bel(S)$, and ν^∞ denotes a suitable "i.i.d. product" of the belief function ν. The de Finetti–Savage model is the special case where (the Independence axiom is satisfied and hence) each ν in the support of μ is additive.

[1] Belief functions are special cases of capacities (or "non-additive probabilities"), sometimes referred to as totally, completely, or infinitely monotone capacities. They originated in Dempster [8]; definitions for more general settings can be found, for example, in Philippe, Debs and Jaffray [34], and Molchanov [33]. See Appendix A for details on belief functions and the corresponding utility functions.