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Carrier fringes by axial translation of the first lens in a double aperture common-path interferometer

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ABSTRACT

This paper presents a method for introducing carrier fringes in a double-aperture common-path interferometer. By using the Fresnel diffraction theory, we demonstrated that the axial displacement of the first lens of the 4*f* optical system involved is linearly proportional to the carrier frequency introduced in the interferogram (both for positive or negative values) in a wide range. Because this displacement is of the order of centimetres, its experimental generation is very simple, practical, and with acceptable accuracy, being carried out by means of a non-sophisticated system. A theoretical model and experimental results are shown.

1. Introduction.

A double-aperture common-path interferometer (DACPI) consists of a 4*f* optical imaging system with two apertures in the input plane and a grating [1] in the Fourier plane, acting as a spatial filter [2–4]. In order to observe the effect of interference in the image plane, the distance between the two apertures must be equalled to the product of the ruling period with the focal length of the lens divided by the wavelength of the employed light. As known, a transversal displacement of the grating produces a phase-step in the interferogram [3,4], and on the other hand, as it was demonstrated recently, an axial displacement of the grating produces carrier fringes [5,6]. More recently, in our previous work [7] we presented another method for introducing carrier fringes in a DACPI, which was based on placing two additional rulings of different periods, one ruling in each input aperture. We demonstrated that the difference of periods is proportional to the carrier frequency introduced in the interferogram. This way of introducing carrier fringes does not need to tilt the beam as typically done with a tilted mirror [8], a wedge prism [9], or a tilted beam-splitter [10]. The generation of carrier fringes is useful for phase extraction as an alternative method of phase-shifting interferometry (PSI). The Fourier-transform of an interferogram with carrier fringes consists of three lobules separated by the value of the introduced carrier frequency. The lobules do not overlap when the carrier frequency is higher than the maximum frequency. Thus, any lobule can be filtered and processed for phase, background and light modulation retrieval. This method is known as carrier fringes interferometry (CFI), and was first proposed by Takeda et al. [11].

In this paper, we propose an easy and practical method for introducing carrier fringes in an interferogram with acceptable accuracy. Because the technique does not generate a tilt between the two beams, the common-path features of the system still remain. This method is obtained from a modified version of a DACPI that consists of displacing axially the first lens from its initial plane while the length 4*f* between the input and image planes is kept constant, as depicted in Fig. 1. In order to show how this carrier is introduced, an analysis based on the Fresnel diffraction theory [12] is applied in order to construct a mathematical model of a properly modified DACPI.

2. Theoretical model

As it can be noted, two modules mounted in cascade form the modified DACPI shown in Fig. 1. It is easy to see that each module is a particular case of the scheme shown in Fig. 2, which consists of two planes and a lens with focal length $f>0$. This lens is placed at a distance z_1 from the object plane or input plane while the output plane or observation plane is placed at a distance z_2 from the lens. So, the distance between these two planes is given by z_1+z_2.

An analysis of this basic system could be made by using the transfer function of the lens given by $\exp[-i(k/2f)(\xi^2+\eta^2)]$ and by calculating the diffracted field for each empty space between two

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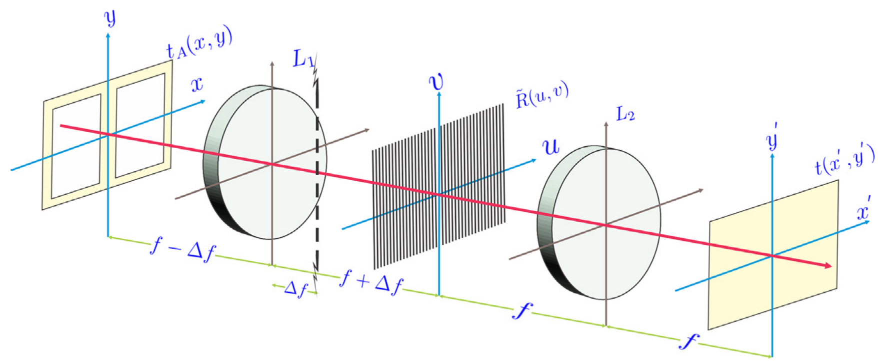

Fig. 1. Modified double-aperture common-path interferometer, carrier fringes are generated when the first lens is placed outside of its initial plane by a distance Δf on axial direction.

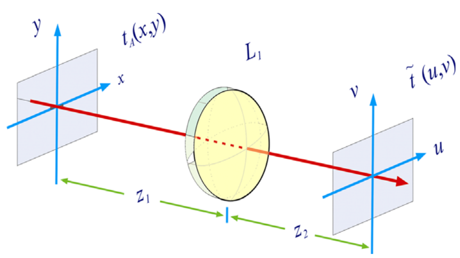

Fig. 2. Elemental optical system formed with a lens and two planes to explain a modified DACPI in Fig. 1.

planes using the Fresnel diffraction theory [8]. This gives,

$$t(\xi,\eta)=\frac{1}{i\lambda z_1}e^{i\frac{k}{2z_1}(\xi^2+\eta^2)}\iint_\infty dxdy\,t_A(x,y)e^{i\frac{k}{2z_1}(x^2+y^2)}e^{-i\frac{k}{z_1}(\xi x+\eta y)} \tag{1}$$

Eq. (1) describes the field at the plane immediately before of the lens, that started as $t_A(x,y)$ at the input plane and travelled a distance z_1, λ is the wavelength of the light, and $k=2\pi/\lambda$ is the wavenumber. Eq. (1) is known as the Fresnel diffraction integral [12]. Then, multiplying the field described in Eq. (1) with the transfer function of the lens and after recalculating the diffraction calculation for the second empty space, it is possible to find that the field at the output plane would be expressed as

$$\tilde{t}(u,v)=\frac{1}{i\lambda f_e}e^{i\frac{k}{2f_e}(1-z_1/f)(u^2+v^2)}\iint_\infty dxdy\,t_A(x,y)e^{i\frac{k}{2f_e}(1-z_2/f)(x^2+y^2)}e^{-i(k/f_e)(xu+yv)} \tag{2}$$

where $f_e=z_1+z_2-z_1z_2/f$. Note that, for $z_1=z_2=f$, it is easy to see that $f_e=f$, and the quadratic phases in Eq. (2) are dropped, so that Eq. (2) is reduced mathematically to the optical Fourier-transform of the transmittance function $t_A(x,y)$. Then, Eq. (2) describes the Fresnel diffraction at the observation plane, which becomes the Fraunhofer diffraction when the observation is made at the focal length of the lens.

Now, if $z_1=f-\Delta f$ and $z_2=f+\Delta f$ are considered for the first module in Fig. 1, then $f_e=f+\Delta f^2/f$ is obtained, and substituting these distances in Eq. (2), this expression is reduced to

$$\tilde{t}(u,v)=\frac{1}{i\lambda}\frac{f}{f^2+\Delta f^2}e^{i(k/2)(\Delta f/f^2+\Delta f^2)(u^2+v^2)}\iint_\infty dxdy\,t_A(x,y)e^{-i(k/2)(\Delta f/f^2+\Delta f^2)(x^2+y^2)}e^{-ik(f/f^2+\Delta f^2)(xu+yv)} \tag{3}$$

which expresses the field just before the Ronchi ruling $\tilde{R}(u,v)$, while, $\tilde{t}(u,v)\tilde{R}(u,v)$ is the field just after the Ronchi ruling, and is considered as the input field for module 2. In this last module, $z_1=z_2=f$ is assumed, then $f_e=f$, and by substituting these distances in Eq. (2) and changing the coordinates accordingly, we have obtained the following expression

$$t(x',y')=\frac{1}{i\lambda f}\iint_\infty dudv\,\tilde{t}(u,v)\tilde{R}(u,v)\times e^{-i(k/f)(ux'+vy')} \tag{4}$$

where $\tilde{R}(u,v)$ has its grating lines running parallel to the v-axis. With a fill factor of $1/2$ and the grating period u_p, the ruling could be modelled as $\tilde{R}(u,v)=rect(2u/u_p)\otimes\sum\delta(u-nu_p)$, where rect(...) is the rectangle function and the symbol $\otimes$ denotes the convolution operation. The same ruling in the form of complex Fourier series could be written as

$$\tilde{R}(u,v)=\frac{1}{2}\sum_n \text{sinc}\left(\frac{1}{2}n\right)e^{i2\pi(n/u_p)u} \tag{5}$$

Substituting Eqs. (3) and (5) into Eq. (4), it is possible to demonstrate after several manipulations that the optical field in the observation plane is given by

$$t(x',y')=\left(1+\frac{\Delta f^2}{f^2}\right)e^{-i\frac{k\Delta f}{2f^2}\left(1+\frac{\Delta f^2}{f^2}\right)(x'^2+y'^2)}\sum_n c_n e^{-i\pi n^2\frac{\lambda\Delta f}{u(2/p)}\left(1+\frac{\Delta f^2}{f^2}\right)}$$
$$e^{i2n\pi\frac{\Delta f}{u_pf}x'}t_A\left(\left(1+\frac{\Delta f^2}{f^2}\right)\left(\frac{\lambda f}{u_p}n-x'\right)-\left(1+\frac{\Delta f^2}{f^2}\right)y'\right) \tag{6}$$

where $2c_n=\text{sinc}(n/2)$ and $1+\Delta f^2/f^2$ is a scale factor that can be approximated to unity since in the experiment the condition $\Delta f \ll f$ is obtained easily. Eq. (6) consists basically of inverted replicas of the input optical disturbance $t_A(x,y)$, where each replica is situated at integer multiples of $\lambda f/u_p$ on x'-axis with its amplitude scaled by the sinc function and a constant phase factor depending on n^2 and Δf, and a linear phase term on x'-direction depending on n and Δf. Observe that if $\Delta f=0$, Eq. (6) is reduced to a known result [1,2,] and Fig. 2 becomes the typical $4f$ optical imaging system.

In order to carry out an interferometric analysis, the transmittance function is considered to be of the form

$$t_A(x,y)=w\left(x+\frac{1}{2}x_0,y\right)E_o+w\left(x-\frac{1}{2}x_0,y\right)E_r \tag{7}$$

where $w(x,y)=rect(x/a_w)rect(y/b_w)$ is a rectangular aperture of sides a_w and b_w, x_0 is the separation distance between two apertures, which are separated symmetrically with respect to origin. Omitting the coordinates of $E_s=A_se^{i\phi_s}$ with $s=o,r$ denoting the probe and reference beams: A_s and ϕ_s are their amplitude