Available online at www.sciencedirect.com

ScienceDirect

Journal of Economic Theory 152 (2014) 105–129

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Altruistically unbalanced kidney exchange ☆

Tayfun Sönmez [1], M. Utku Ünver [1]

Department of Economics, Boston College, 140 Commonwealth Ave., Chestnut Hill, MA 02467, United States

Received 8 February 2012; final version received 30 June 2013; accepted 27 February 2014

Available online 16 April 2014

Abstract

Although a pilot national live-donor kidney exchange program was recently launched in the US, the kidney shortage is increasing faster than ever. A new solution paradigm is able to incorporate *compatible pairs* in exchange. In this paper, we consider an exchange framework that has both compatible and incompatible pairs, and patients are indifferent over compatible pairs. Only two-way exchanges are permitted because of institutional constraints. We explore the structure of Pareto-efficient matchings in this framework. We show that under Pareto-efficient matchings, the same number of patients receive transplants, and it is possible to construct Pareto-efficient matchings that match the same incompatible pairs while matching the least number of compatible pairs. We extend the celebrated Gallai–Edmonds Decomposition in the combinatorial optimization literature to our new framework. We also conduct comparative static exercises on how this decomposition changes as new compatible pairs join the pool.

JEL classification: C78; D78; D02; D63

Keywords: Kidney exchange; Market design; Matching

☆ We would like to thank Marek Pycia, Alvin E. Roth, Itai Sher, and Özgür Yilmaz for comments. Tayfun Sönmez and M. Utku Ünver acknowledge the research support of NSF (SES-0616470, SES-0616689, and CMMI-1069100). Sönmez acknowledges the research support of Goldman Sachs Gives via Dalinc Ariburnu – Goldman Sachs Faculty Research Fund. We would also like to thank participants at Villa La Pietra Applied Microeconomics Conference '11 at NYU Florence, CoED '11 at Montreal; PET '11 at Indiana; Johns Hopkins; BQGT '10 at Newport Beach, CA; Market Design Conference at Yonsei University, Korea; SAET '09 at Ischia, Italy; GAMES '08 at Northwestern; and Market Design Conference '08 at MIT. We also thank an associate editor and a referee for their useful comments.

E-mail addresses: sonmezt@bc.edu (T. Sönmez), unver@bc.edu (M.U. Ünver).

URLs: http://www.tayfunsonmez.net (T. Sönmez), http://www2.bc.edu/~unver (M.U. Ünver).

[1] Distinguished Research Fellow, Koç University.

http://dx.doi.org/10.1016/j.jet.2014.04.003

1. Introduction

In the last decade, market design found an unexpected practical application in kidney exchange, which led to an interdisciplinary collaboration between economists and medical professionals to establish several kidney exchange programs. To explain and motivate the contribution of the current paper, it is essential to describe how this collaboration has evolved over the years, and led to new innovations in kidney exchange. In the early 2000s, economists observed that the two main types of kidney exchanges conducted in the US correspond to the most basic forms of exchanges in a house allocation model [1]. Building on this setup, they formulated a kidney exchange model and proposed a *top trading cycles and chains* mechanism (TTCC) (Roth, Sönmez, and Ünver, henceforth, RSÜ [15]). In their simulations RSÜ [15] have shown that, in contrast to the 45 percent of the patients with willing live donors who fail to receive a transplant in the absence of kidney exchanges, fewer than 10 percent would remain without a transplant under TTCC.

When economists shared their findings with the medical community, two reservations were expressed regarding RSÜ [15]. First of all, RSÜ [15] allowed for potentially large exchanges that would be logistically hard to implement since all transplants in an exchange need to be carried out simultaneously. The second concern was that RSÜ [15] assumed strict preferences between compatible kidneys, which is contrary to the general tendency in the US where doctors assume that two compatible living-donor kidneys have essentially the same survival rates [8,3].

To address these concerns, RSÜ [16] proposed a second model that restricted the size of kidney exchanges to two patient-donor pairs and assumed that patients are indifferent between compatible kidneys. RSÜ [16] observed that their *pairwise kidney exchange* model is an application of a well-analyzed problem in the discrete-optimization literature,[2] some of the techniques of which were recently imported to economic theory by Bogomolnaia and Moulin [2] for two-sided matching markets.[3] The optimal-matching methodology proposed by RSÜ [16] became the basis of practical kidney exchange throughout the world including at the New England Kidney Exchange Program (NEPKE) – the first exchange program using an optimization-based mechanism – and the Alliance for Paired Donation (APD), both of which were formed as a result of the collaboration between economists and medical professionals. Most recently, the National Kidney Paired Donation Pilot Program in the US and National Program in the UK were established based on similar principles.[4]

An earlier, abstract version of the RSÜ [16] model was analyzed extensively in the 1960s. One of the most important contributions to this literature was that of Gallai [5,6] and Edmonds [4], who characterized the set of Pareto-efficient matchings. This result is known as the *Gallai–Edmonds Decomposition* (GED) Theorem, and it plays a central role in our current paper. One of the corollaries to the GED Theorem has a very plausible implication for pairwise kidney exchange: the same number of patients are matched at every Pareto-efficient matching. Hence, a program never matches a high-priority patient at the expense of multiple patients under the Pareto-efficient *pairwise priority mechanisms* offered by RSÜ [16]. This result does not hold for TTCC or more generally for mechanisms that allow larger exchanges than pairwise. Hence, from a medical ethics perspective it gives pairwise priority mechanisms an edge. However, this

[2] See Lovász and Plummer [11] and Korte and Vygen [10] for comprehensive surveys of this literature.

[3] See Yılmaz [21] for an application of this two-sided matching approach in kidney exchange.

[4] These four programs also allow for three-way exchanges based on findings of RSÜ [18] and Saidman et al. [19], see below.