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Stochastic fictitious play with continuous action sets

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Abstract

Continuous action space games are ubiquitous in economics. However, whilst learning dynamics in normal form games with finite action sets are now well studied, it is not until recently that their continuous action space counterparts have been examined. We extend stochastic fictitious play to the continuous action space framework. In normal form games with finite action sets the limiting behaviour of a discrete time learning process is often studied using its continuous time counterpart via stochastic approximation. In this paper we study stochastic fictitious play in games with continuous action spaces using the same method. This requires the asymptotic pseudo-trajectory approach to stochastic approximation to be extended to Banach spaces. In particular the limiting behaviour of stochastic fictitious play is studied using the associated smooth best response dynamics on the space of finite signed measures. Using this approach, stochastic fictitious play is shown to converge to an equilibrium point in two-player zero-sum games and a stochastic fictitious play-like process is shown to converge to an equilibrium in negative definite single population games.

JEL classification: C72; C73; D83; C14

Keywords: Stochastic fictitious play; Learning in games; Continuous action set games; Abstract stochastic approximation

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1. Introduction

Continuous action space games are ubiquitous in economics. However whilst learning dynamics in normal form games with finite action sets are now well studied (e.g. [18]) it is not until recently that their continuous action space counterparts have been examined. Oechssler and Riedel [39] and Lahkar and Riedel [31] provide existence and uniqueness results for two of the most commonly studied evolutionary dynamics: the replicator dynamics and logit best response dynamics, in the single population scenario. Further results along similar lines are given by Oechssler and Riedel [40], Seymour [41], Cressman [12], Cressman et al. [13] and Hofbauer et al. [23].

Although these dynamics have been studied in continuous time for games with continuous action spaces there are few existing convergence results for discrete time learning. Hofbauer and Sorin [22] study a fictitious play-like process in which each player best responds to the average action played by their opponent(s) rather than their opponent(s) empirical distribution(s). They show that this fictitious play-like process converges to a global attractor of the associated best response dynamic in two-player zero-sum continuous action games. Alternatively, Chen and White [11] investigate a stochastic fictitious play-like model which would be difficult to implement (see Section 3.2 for a discussion) which assumes each player uses a probability density function to represent their beliefs. This rules out placing positive mass on a particular observed action, which would correspond to a Dirac measure, and therefore does not have an associated density.

We study a stochastic fictitious play process in continuous action space games, in which beliefs are the empirically observed action distribution. This requires development of new stochastic approximation tools to contend with the resulting measure-valued process.

Dynamical systems results of Lahkar and Riedel [31], combined with our stochastic approximation theory, allow us to analyse a variant of stochastic fictitious play in single-population games. We prove convergence of the stochastic fictitious play-like process in negative definite single population games with a continuous action set and bounded, Lipschitz continuous rewards.

Furthermore we extend the results of Lahkar and Riedel [31] to the N-player case. This allows us to analyse stochastic fictitious play in N-player continuous action space games. We prove the global convergence of the logit best response dynamics for two-player zero-sum games with continuous action spaces and bounded, Lipschitz continuous rewards. Convergence of stochastic fictitious play follows by applying our stochastic approximation results. This extends the previous results of Fudenberg and Kreps [17], Benaïm and Hirsch [2], Hofbauer and Sandholm [21] and Hofbauer and Hopkins [20] for stochastic fictitious play in normal form games with finite action sets.

This paper is organised in the following manner. Section 2 introduces the formal model which we consider. Section 3 contains essential background material from Benaïm [1] and an extension to this asymptotic pseudo-trajectory approach to stochastic approximation on a Banach space. In Section 4 we analyse the stochastic fictitious play process. We develop the logit best response dynamic for N-player, continuous action space games, and show convergence to the set of logit equilibria in two-player zero-sum games with continuous action sets and bounded, Lipschitz continuous rewards. Using the stochastic approximation framework of Section 3 we show that stochastic fictitious play converges to an equilibrium for two-player zero-sum games with continuous action sets and bounded, Lipschitz continuous rewards. In Section 5 the work of Lahkar and Riedel [31] covering the logit best response dynamic for single population, continuous action games is reviewed and a learning variant of this, similar to stochastic fictitious play, is studied. As in the N-player case, this stochastic fictitious play variant is shown to converge in single