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# The Gini coefficient: Majority voting and social welfare ☆

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**Abstract**

Majority voting and social evaluation functions are the main alternatives proposed in the literature for aggregating individual preferences. Despite these being very different, this paper shows that the ranking of income distributions, symmetric under the same transformation, by S-Gini consistent social evaluation functions and majority voting coincide if and only if the inequality index under consideration is the Gini coefficient. In this case, we show that the equally distributed equivalent income is equal to the median of the distribution. In addition, we find that the Gini coefficient is just an affine function of the median–mean ratio.



*JEL classification:* D31; D63; P16

*Keywords:* Social evaluation function; The Gini coefficient; Majority voting; Median–mean ratio; Symmetric distribution

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## 1. Introduction

Traditionally, two strategies have been adopted by scholars to aggregate individual preferences and, hence, to rank distributions: 1) a political process like the majority voting mechanism (see, among others, Black [5], Romer [28] and Bishop et al. [4]); and 2) a social evaluation function (SEF) derived from a set of "desirable" assumptions (Kolm [20], Atkinson [3] and Blackorby et al. [7]). The first procedure, majority voting, is the binary decision rule most commonly used in decision-making bodies, and involves selecting the distribution that receives more than half of the votes. Meanwhile, a social evaluation function provides the set of axioms that has to be assumed in order to reach a particular social decision. Despite their evident differences, the two approaches have recently been linked. Salas and Rodríguez [30] have shown that for those distributions that are symmetric under the same strictly increasing transformation, the Atkinson–Kolm–Sen (AKS) class of utilitarian social evaluation functions (Kolm [20], Atkinson [3] and Sen [32]), consistent with the Kolm-Atkinson index of inequality, accords with the majority voting procedure.

In principle, the extension of this result to the class of rank-dependent SEFs consistent with the widely used Gini coefficient is problematic given the result in Newbery [26]. This author found that there is no differentiable strictly concave utility function such that a utilitarian SEF $W$ accords with the Gini coefficient. Worse still, Dasgupta et al. [10] generalized Newbery's result from $W$ to any strictly quasi-concave SEF and, later on, Lambert [23] directly generalized Newbery's result from $W$ to any differentiable SEF.

Fortunately, some authors (see Sheshinski [33], Sen [32], Kakwani [18] and Lambert [23]) have argued that a convincing rationale for the use of a SEF consistent with the Gini coefficient could still exist if we abandon the class of individualistic social evaluation functions.[1] In particular, Kakwani [18] and Lambert [23] provided a positive result by widening the domain for personal preferences to incorporate envy or altruism.

In this paper we link the class of Kakwani–Lambert (KL) SEFs (Kakwani [18] and Lambert [23]) consistent with the class of S-Gini indices and majority voting. For this purpose, we first look for the transformation that makes the equally distributed equivalent income (EDE) of this class of rank-dependent KL SEFs equal to the median income. The transformation turns out to be not increasing with the rank. And we then show that majority voting and the class of rank-dependent KL SEFs are consistent if and only if the inequality index under consideration is the Gini coefficient.

A number of political economy models proposed in the literature rely on the (inverse) relationship between inequality and the median–mean ratio (see, for example, Meltzer and Richard [25] and Alesina and Rodrik [2]). However, no theoretical proof of this link has been provided. A by-product of this paper is the result that the Gini coefficient can be written as a simple affine decreasing function of the median–mean ratio. Thus, the widely used Gini coefficient can be summarized by two common measures of position, namely the mean and median values.

The structure of the paper is as follows. In Section 2, we provide our main results for the class of rank-dependent KL SEFs. Section 3 discusses the proposed transformation and comments on the potential limitations of the approach, while Section 4 presents some concluding remarks.

[1] In 1978, Blackorby and Donaldson [6] set out a list of properties that characterized (although not completely) the social evaluation functions which accord with the Gini coefficient. They should be homothetic, quasi-concave and additive but not separable. In 2001, Aaberge [1] fully characterized the preference orderings related to the Gini coefficient and to the extended S-Gini coefficients.