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Journal of Economic Theory 152 (2014) 237–248

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Transitive regret over statistically independent lotteries ☆

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Received 14 May 2012; final version received 16 December 2013; accepted 27 February 2014

Available online 6 May 2014

Abstract

Preferences may arise from regret, i.e., from comparisons with alternatives forgone by the decision maker. When each outcome in a random variable is compared with the parallel outcome in an alternative random variable, regret preferences are transitive iff they are expected utility. In this paper we show that when the choice set consists of pairwise statistically independent lotteries and the regret associated with each outcome is with respect to the entire alternative distribution, then transitive regret-based behavior is consistent with betweenness preferences and with a family of preferences that is characterized by a consistency property. Examples of consistent preferences include CARA, CRRA, and anticipated utility.

JEL classification: D81

Keywords: Regret; Transitivity; Non-expected utility

1. Introduction

One of the most convincing psychological alternatives to expected utility theory is regret theory, which was independently developed by Bell [1] and by Loomes and Sugden [8]. The basic form of the theory applies to choices made between pairs of random variables. While in Savage's [13] model the decision maker evaluates a random variable by weighting its outcomes,

☆ We thank an anonymous referee for helpful comments and suggestions.
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http://dx.doi.org/10.1016/j.jet.2014.05.001
0022-0531/

regret theory suggests that the decision maker should also take into consideration the alternative outcome from the other random variable. This comparison may cause rejoicing — if the actual outcome is better than the alternative — or regret.

Both Bell and Loomes and Sugden assumed that the evaluation of the regret should be additive. That is, for two random variables $X = (x_1, s_1; \dots; x_n, s_n)$ and $Y = (y_1, s_1; \dots; y_n, s_n)$,

$$X \succeq Y \quad \Longleftrightarrow \quad \sum_i p(s_i)\psi(x_i, y_i) \geqslant 0$$

where $p(s_i)$ is the probability of event s_i and ψ is a regret function. If $\psi(x, y) = u(x) - u(y)$ then this theory reduces to expected utility theory and it is easy to verify that unless this is the case, such preferences cannot be transitive. One may suspect the restrictive additive form to be the source of intransitive cycles, but as is proved in Bikhchandani and Segal [2], intransitivity is built into the regret model: even when one adopts the more general form

$$X \succeq Y \quad \Longleftrightarrow \quad V\big(\psi(x_1, y_1), p(s_1); \dots; \psi(x_n, y_n), p(s_n)\big) \geqslant 0, \tag{1}$$

where V is any increasing (with respect to first-order stochastic dominance) functional, transitivity still implies expected utility.[1]

Regret theory can be used to compare statistically independent lotteries (see Loomes and Sugden [8]), where the regret felt upon winning x_i and not y_j is weighted by $p_i q_j$. But consider a gambler who chooses to play the Roulette instead of Craps. While betting on Black in Roulette when the outcome turns out Red, it seems unnatural that he will compare this outcome to each specific roll of the dice in a Craps game he did not play (and probably did not even observe). Rather, it may drive him to regret the fact that he did not play Craps, and the regret is with respect to the whole alternative distribution. Such feelings are the topic of the present paper.

We discuss a choice problem between two statistically independent lotteries $X = (x_1, p_1; \dots; x_n, p_n)$ and $Y = (y_1, q_1; \dots; y_m, q_m)$. When evaluating the lottery X, the decision maker forecasts his ex-post feelings, and considers his regret or rejoice when he will know that he won x_i but did not play the lottery Y.[2] Formally, we analyze binary relations that are defined by $X \succeq Y$ iff

$$V\big(\psi(x_1, Y), p_1; \dots; \psi(x_n, Y), p_n\big) \geqslant 0 \geqslant V\big(\psi(y_1, X), q_1; \dots; \psi(y_m, X), q_m\big) \tag{2}$$

where $\psi(x, Y)$ is the rejoice or regret felt by the decision maker upon learning that he won x in lottery X which he chose to play out of the set $\{X, Y\}$.[3] We call this property *distribution regret*. The question we ask is this: Under what conditions are distribution-regret relations transitive?

Unlike [2], where it was shown that only expected utility preferences are consistent with both Eq. (1) and transitivity, here we identify two families of preferences that satisfy distribution regret, i.e. Eq. (2), and transitivity. The first are betweenness preferences, according to which $X \succeq Y$ iff for all $\alpha \in [0, 1]$, $X \succeq \alpha X + (1 - \alpha)Y \succeq Y$ (see Chew [3,4], Fishburn [7], and

[1] If $\succeq$ is defined on the set of independent lotteries then transitivity and (1) do not imply expected utility. See Machina [10, footnote 20].

[2] These feelings do not have to agree with his initial preferences over lotteries. That is, at this stage we do not rule out the possibility of preference for the outcome x_i over Y together with anticipated regret if X is chosen and x_i is drawn. But this will be ruled out by transitivity.

[3] Note that if a preference relation over independent lotteries satisfies Eq. (1) in a linear form then it also satisfies Eq. (2) in a linear form. Such a preference relation was suggested without a further discussion by Machina [10] and by Starmer [14]. We provide additional results in Section 3 below.