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Purely subjective Maxmin Expected Utility [☆,☆☆]

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Abstract

The Maxmin Expected Utility decision rule suggests that the decision maker can be characterized by a utility function and a set of prior probabilities, such that the chosen act maximizes the minimal expected utility, where the minimum is taken over the priors in the set. Gilboa and Schmeidler axiomatized the maxmin decision rule in an environment where acts map states of nature into simple lotteries over a set of consequences. This approach presumes that objective probabilities exist, and, furthermore, that the decision maker is an expected utility maximizer when faced with risky choices (involving only objective probabilities). This paper presents axioms for a derivation of the maxmin decision rule in a purely subjective setting, where acts map states to points in a connected topological space. This derivation does not rely on a pre-existing notion of probabilities, and, importantly, does not assume the von Neumann and Morgenstern (vNM) expected utility model for decision under risk. The axioms employed are simple and each refers to a bounded number of variables.

JEL classification: D81

Keywords: Maxmin Expected Utility; Purely subjective probability; Uncertainty aversion; Tradeoff consistency; Biseparable preference

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1. Introduction

There is a respectable body of literature dealing with axiomatic foundations of decision theory, and specifically with the non-Bayesian (or extended Bayesian) branch of it. We will mention a part of this literature to put our paper in context.

Building on the works of Ramsey [25], de Finetti [9], and von Neumann and Morgenstern [24], Savage [26] provided an axiomatic model of purely subjective expected utility maximization. The descriptive validity of this model was put in doubt long ago. Today many think that Savage's postulates do not constitute a sufficient condition for rationality, and some doubt that they are all necessary conditions for it. However, almost all agree that his work is by far the most beautiful and important axiomatization ever written in the social or behavioral sciences. "The crowning glory", as Kreps [22, p. 120] put it. Savage's work has had a tremendous influence on economic modeling, convincing many theorists that the only rational way to make decisions is to maximize expected utility with respect to a subjective probability. Importantly, due to Savage's axioms, many believe that any uncertainty can and should be reduced to risk, and that this is the only reasonable model of decision making on which economic applications should be based. About a decade after Savage's seminal work Anscombe and Aumann [3] (AA for short) suggested another axiomatic derivation of subjective probability, coupled with expected utility maximization. As in Savage's model, acts in AA's model map states of nature to a set of consequences. However, in Savage's model the set of consequences has no structure, and it may consist of merely two elements, whereas AA assume that the consequences are lotteries as in vNM's model, namely, distributions over a set of outcomes, whose support is finite. Moreover, AA impose the axioms of vNM's-theory (including the independence axiom) on preferences over acts, which imply that the decision maker maximizes expected utility in the domain of risk. On the other hand, AA's model can deal with a finite state space, whereas Savage's axioms imply that there are infinitely many states, and, moreover, that none of them is an "atom". Since in many economic applications there are only finitely many states, one cannot invoke Savage's theorem to justify the expected utility hypothesis in such models.

A viable alternative to the approaches of Savage and of AA is the assumption that the set of consequences is a connected topological space. (See Fishburn [11] and Krantz, Luce, Suppes, and Tversky [21] and the references therein.) These spaces are "rich" and therefore more restrictive than Savage's abstract set of consequences. On the other hand, such spaces are natural in many applications. In particular, considering a consumer problem under uncertainty the consequences are commodity bundles which, in the tradition of neoclassical consumer theory, constitute a convex subset of an n-dimensional Euclidean space, and thus a connected topological space. As opposed to AA's model, the richness of the space is not necessarily derived from mixture operations on a space of lotteries. Thus, no notion of probability is pre-supposed, and no restrictions are imposed on the decision maker's behavior under risk.

Despite the appeal of Savage's axioms, the Bayesian approach has come under attack on descriptive and normative grounds alike. In accord with the view held by Keynes [18], Knight [19], and others, Ellsberg [10] showed that Savage's axioms are not necessarily a good description of how people behave, because people tend to prefer known to unknown probabilities. Moreover, some researchers argue that such preferences are not irrational. This was also the view of Schmeidler [27], who suggested the first axiomatically based, general-purpose model of decision making under uncertainty, allowing for a non-Bayesian approach and a not necessarily neutral attitude to uncertainty. Schmeidler axiomatized expected utility maximization with a nonadditive probability measure (also known as capacity), where the operation of integration is done

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