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Delayed-response strategies in repeated games with observation lags

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Abstract

We extend the folk theorem of repeated games to two settings in which players' information about others' play arrives with stochastic lags. In our first model, signals are almost-perfect if and when they do arrive, that is, each player either observes an almost-perfect signal of period-t play with some lag or else never sees a signal of period-t play. The second model has the same lag structure, but the information structure corresponds to a lagged form of imperfect public monitoring, and players are allowed to communicate via cheap-talk messages at the end of each period. In each case, we construct equilibria in "delayed-response strategies," which ensure that players wait long enough to respond to signals that with high probability all relevant signals are received before players respond. To do so, we extend past work on private monitoring to obtain folk theorems despite the small residual amount of private information.

JEL classification: C73

Keywords: Repeated games; Folk theorem; Private monitoring; Observation lag

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1. Introduction

Understanding when and why individuals cooperate in social dilemmas is a key issue not just for economics but for all of the social sciences,[1] and the theory of repeated games is the workhorse model of how and when concern for the future can lead to cooperation even if all agents care only about their own payoffs. The clearest expression of this idea comes as players become arbitrarily patient; here various folk theorems provide conditions under which approximately efficient payoffs can be supported by equilibrium strategies. Because these results are so influential, it is important to understand which of their assumptions are critical and which are merely convenient simplifications; a large literature (discussed below) has extended the folk theorems under successively weaker assumptions about the "monitoring structures" that govern the signals players receive about one another's actions.

Here we relax an assumption which is maintained throughout most of the prior repeated games literature: the assumption that signals of the actions taken in each period (simultaneously) arrive immediately after players' actions in that period. We consider repeated games in which the players' signals about other player's actions arrive with stochastic and privately-observed lags. Our folk theorems for settings with lagged signals show that the assumption that signals are observed immediately is not necessary for repeated play to support cooperation.

To prove these folk theorems, we use the idea of "delayed-response" strategies, under which players wait to respond to signals of a given period's play for long enough that it is likely (although not certain) that every player has observed the relevant signals by the time players respond to signal information. Although the observation lags generate a form of imperfect private monitoring, the private information here has a special form that allows delayed-response strategies to construct the same set of limit equilibrium payoffs as if the lags were not present.

More specifically, we suppose that players act simultaneously each period, and that players' actions jointly determine a probability distribution over signals, but that players

- do not observe signals immediately and
- might observe signals asynchronously.

The times at which observation occurs are private information and may be infinite, that is, a particular signal may never arrive. Some sort of observation lags seem plausible in many cases; for example there may be a small probability that a player is momentarily inattentive and temporarily does not see his or her partner's actions; more strongly, in some cases a player may never learn just what happened during moments of inattention. Moreover, information lags of multiple periods seem especially appropriate in settings for which the time period under consideration is extremely short (Fudenberg and Levine [14,16] and Sannikov and Skrzypacz [34]), and in continuous-time models, where the "period length" is effectively 0 (Bergin and MacLeod [6], Sannikov [32], Sannikov and Skrzypacz [33], and Faingold and Sannikov [12]).[2]

To prove our folk theorems, we construct *delayed-response strategies*, in which the repeated game is divided into a finite number of "threads," with play in each thread independent of play in the other threads. Section 3 examines the simplest application of this idea, which is to the case of bounded lags, where there is a K such that every signal arrives within K periods of play. In that setting, using strategies that have $K + 1$ threads, we can ensure that each thread is

[1] See e.g., the work of Ahn et al. [3], Gachter et al. [21].

[2] Indeed, physics suggests that the speed of light is a constraint on the speed with which signals can travel.