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A two-parameter model of dispersion aversion

Robert G. Chambers [a], Simon Grant [b,*], Ben Polak [c], John Quiggin [b]

[a] Department of Agricultural and Resource Economics, University of Maryland, United States
[b] School of Economics, University of Queensland, Australia
[c] Office of the Provost, Yale University, United States

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Abstract

The idea of representing choice under uncertainty as a trade-off between mean returns and some measure of risk or uncertainty is fundamental to the analysis of investment decisions. In this paper, we show that preferences can be characterized in this way, even in the absence of objective probabilities. We develop a model of uncertainty averse preferences that is based on a mean and a measure of the dispersion of the state-wise utility of an act. The dispersion measure exhibits positive linear homogeneity, sub-additivity, translation invariance and complementary symmetry. Since preferences are only weakly separable in terms of these two summary statistics, the uncertainty premium need not be constant. We generalize the concept of decreasing absolute risk aversion. Further we derive two-fund separation and asset pricing results analogous to those that hold for the standard CAPM.

JEL classification: D81; G11; G12

Keywords: Uncertainty aversion; Mean utility; Dispersion of utility; Weak-separability; Two-fund separation; CAPM excess return formula

1. Introduction: dispersion versus return

Ever since the pioneering work of Markowitz [18] and Tobin [24], the idea of representing investment decisions in terms of a trade-off between risk (often characterized by some measure of the dispersion or variation of the return) and expected return has played a prominent role

* Corresponding author.
E-mail addresses: rchambers@arec.umd.edu (R.G. Chambers), simongrant@uq.edu.au (S. Grant), benjamin.polak@yale.edu (B. Polak), j.quiggin@uq.edu.au (J. Quiggin).

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in finance theory. The mean-variance analysis presented by Markowitz and Tobin formed the basis of the Capital Asset Pricing Model (CAPM) (Sharpe [22], Lintner [14]) which remains the main workhorse of financial analysis. However, mean-variance analysis has been subject to a wide range of criticisms. The first criticism came from proponents of expected utility theory (EUT), who observed that mean-variance analysis was consistent with EUT only for the special (and unattractive) case of a quadratic utility function.[1] If the EUT hypothesis is abandoned, it is possible to consider more general mean-variance preferences, but these are typically ad hoc functional forms, lacking the axiomatic foundations that characterize EUT.

A more recent set of criticisms relates to the choice of the variance or equivalently, the standard deviation as the measure of risk. While the standard deviation has appealing qualities,[2] a large body of evidence suggests that the return distributions for many assets are 'fat-tailed' having excess kurtosis relative to the normal. This suggests the need either to take higher moments into account, which substantially complicates the analysis, or to use measures of riskiness other than the standard deviation.

More fundamental criticisms arise from the work of Ellsberg [4]. Mean-variance analysis typically treats probabilities as if they are objectively known, or at least as if they can be derived from observed preferences as in Savage [21]. But there is ample evidence to suggest that many decision-makers do not display preferences consistent with well-defined subjective probabilities (probabilistic sophistication in the terminology of Machina and Schmeidler [17]). In particular, preferences may display source dependence as in Chew and Sagi [3] or Ergin and Gul [6]. Decision-makers may prefer either side of a symmetric bet that is well-understood, such as a coin toss, over either side of an apparently symmetric bet on an unfamiliar event, such as up or down daily movements in temperature in an unfamiliar city.[3]

In this paper, we address all of these issues. We provide a rigorous foundation for preferences characterized by two arguments, a mean value and a dispersion parameter. The properties of the dispersion parameter generalize those of the standard deviation and are satisfied (modulo an appropriate normalization in some cases) by many of the commonly used measures of dispersion in the statistical literature. Our approach, however, encompasses choice under risk (known objective probabilities), choice under uncertainty (subjective probabilities as in Savage [21]) and choice under ambiguity (where different 'sources' of uncertainty need not be treated symmetrically).

2. Background

In Grant and Polak [12] two of the co-authors of the present paper examine the family of mean-dispersion preferences that admit a representation that takes the form of a 'mean' minus a 'dispersion measure' of the state-contingent utility vector associated with an act. In particular, for these preferences, one can show that there exist an affine utility function U over consequences, a probability weighting vector π on the states and a function ρ over state-utility vectors such that the preferences over acts are represented by the functional

$$V(f) = \mathrm{E}_\pi(U \circ f) - \rho(U \circ f), \tag{1}$$

[1] For example, expected utility with quadratic utility implies that risk preferences exhibit increasing absolute risk aversion.

[2] These are most evident for the case of normal distributions, which are fully characterized by the mean and standard deviation.

[3] Such a source dependence preference may be invoked to explain the phenomenon of home market bias in investment decisions (French and Poterba [9]).