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Existence and non-existence in the moral hazard problem ☆

Sofia Moroni [b,*], Jeroen Swinkels [a]

[a] *Kellogg School of Management, Northwestern University, Evanston, IL, United States*
[b] *Department of Economics, Yale University, New Haven, CT, United States*

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Abstract

We provide a new class of counter-examples to existence in a simple moral hazard problem in which the first-order approach is valid. In contrast to the Mirrlees example, unbounded likelihood ratios on the signal technology are not central. Rather, our examples center around the behavior of the utility function as utility diverges to negative infinity. For any utility function, such as $\ln(w)$, in which utility diverges to negative infinity at a finite wealth level, existence will fail for some specifications of the agent's cost of effort. When utility diverges to negative infinity only as wealth does as well, existence holds for all specifications of the agent's cost of effort if and only if the agent continues to dislike risk as wealth diverges to negative infinity. When there is a finite lower bound on utility, existence is assured. For those cases where existence fails, we characterize the limit of near optimal contracts.

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1. Introduction

Mirrlees [5] provides a classic example of a moral hazard problem with no optimal solution. In the example, which relies on an unbounded likelihood ratio, there is a sequence of contracts

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* Corresponding author. Fax: +1 203 436 9023.
E-mail addresses: sofia.moroni@yale.edu (S. Moroni), j-swinkels@kellogg.northwestern.edu (J. Swinkels).

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that have cost converging to the full information first best, but no contract that achieves the first best.

Jewitt, Kadan, and Swinkels [4] (henceforth JKS) provide a proof of existence when likelihood ratios are bounded. We point to an implicit, and not always correct, assumption in [4] by exhibiting a new class of examples in which there is non-existence of the optimal contract even if the likelihood ratio is bounded. The problem in these examples is that we know that an optimal contract must be of the form given by Holmström [2]. But, in some settings, the principal can easily run out of room to provide adequate incentives by contracts of this form. In particular, we show conditions under which any contract of the required form that gives the right utility provides inadequate incentives.

When utility diverges at a finite consumption level the existence problem is severe – for any given utility function and information structure one can find a specification of the agent's cost of effort and reservation utility such that an optimal contract does not exist. Thus for example, existence is not in fact guaranteed in the workhorse example of log utility.

When utility diverges to $-\infty$ only as consumption does as well, we show a necessary and sufficient condition for the missing step for the proof in [4] to hold, and hence for existence. The condition is that, in a sense to be made precise, the agent remains risk averse as consumption diverges to $-\infty$. Under a technical regularity condition, the condition fails only if u has bounded slope, and so becomes essentially linear as consumption diverges. Hence, the existence problem is not severe for this case. For settings where the utility is bounded below, the original JKS proof goes through without problem.

Because our setting has bounded likelihood ratios, we show that unlike in the Mirrlees example we cannot approach the first best.[1] But, we provide a characterization of both the limit to which any sequence of contracts with costs approaching the second best must converge, and the limit cost. We also show that one can approach the second best using contracts that bound utility strictly above $-\infty$.[2] The limit contract has an intuitive form. It can be thought of as starting from a contract of the standard form that provides both too much utility and too little incentives, and then modifying it by providing very low utility on a small interval near the lowest signal, which both eliminates the extra utility the base contract provides and relaxes the incentive constraint so as to restore feasibility. Since the problem is too much utility and too little incentives, and since the trade-off of extra incentives per unit of utility taken away is most favorable at the lowest signal, the "right" way to do this is to concentrate the modification arbitrarily close to the lowest signal, and it is this that leads to non-existence.

2. Model

The model is standard. A risk neutral principal employs a risk averse agent. The agent's utility function over final wealth is $u : D \to \mathbb{R}$ where u is in C^2 with $u' > 0$ and $u'' < 0$. To focus attention, we assume $D \subseteq \mathbb{R}$ is an interval with upper bound ∞, and lower bound $\underline{d}$, where $\underline{d}$ may

[1] In the case of unbounded likelihood ratios, one can concentrate punishments at outcomes which are very unlikely at the desired effort level, but the probability of which rises fast with a deviation. Here, the only way to provide incentives is to face the agent with risk at the desired action.

[2] The case where utility has a lower bound above $-\infty$ is one for which JKS's existence proof is valid, and for which contracts have a simple characterization as truncated versions of standard Holmström [2] contracts.