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Saddle functions and robust sets of equilibria

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Abstract

We provide a new sufficient condition for the robustness of sets of equilibria to incomplete information in the sense of Kajii and Morris (1997) [11], Morris and Ui (2005) [15]. The condition is formulated for games with a saddle function. A saddle function is a real-valued function on the set of action profiles such that there is a single player for whom *minimizing* the function implies choosing her best response, and for the other players *maximizing* the function implies choosing their best responses. In a game with a saddle function the set of correlated equilibria that induce an expectation of the saddle function greater or equal to its *maximin* value is robust to incomplete information.

JEL classification: C72; D82

Keywords: Incomplete information; Robustness; Potential; Team-maximin equilibrium

1. Introduction

We often model a strategic situation as a complete information game. However, an equilibrium outcome of a complete information game may differ from the outcomes of an arbitrarily "close" incomplete information game [4,18]. This leads Kajii and Morris [11] to introduce

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an equilibrium robust to incomplete information and then Morris and Ui [15] to generalize it to robust sets of equilibria. A set of equilibria of a complete information game is robust if every incomplete information game close to the original game has a Bayesian Nash equilibrium that induces an observed behavior close to some equilibrium in the set. If a robust set is a singleton, it is robust to incomplete information in the sense of Kajii and Morris [11].

Our main contribution is a new sufficient condition for the robustness of sets of correlated equilibria. To provide the condition we first introduce a saddle function of a complete information game. A saddle function is a real-valued function on the set of action profiles such that there is a single player for whom minimizing the function implies choosing her best response, and for the other players maximizing the function implies choosing their best-responses. The value of a saddle function is the *maximin* attained when the saddle function is maximized over strategy profiles of the maximizing players and is minimized over strategies of the minimizing player. We show that the set of correlated equilibria that induce an expectation of a saddle function greater or equal to the value is robust.

Our condition is applicable to several interesting classes of games. One special class is team vs. adversary games, which are zero-sum games where a group of players with identical payoffs plays against a single adversary, studied by von Stengel and Koller [25]. While team vs. adversary games are rather restrictive, a saddle function exists in a broader class of games "strategically equivalent" to team vs. adversary games. To demonstrate the scope of economic applications we develop a simple model of private provision of defense in line with the literature on attacker–defender games [2,3,5,6,16].[1]

The literature on the robustness to incomplete information stems from Kajii and Morris [11], who introduce the notion and give sufficient conditions in terms of a unique correlated equilibrium and **p**-dominant equilibrium. The former condition implies that a unique Nash equilibrium of a two-person zero-sum game is robust. Next, Ui [23] proves the robustness of a unique maximizer of a potential function defined by Monderer and Shapley [13], and Tercieux [21] obtains a condition for games with **p**-best-response sets. Morris and Ui [15] provide a sufficient condition in terms of generalized potential functions which generalizes the above approaches, but is difficult to apply. They also develop tractable conditions in terms of special classes of generalized potential functions: best-response potentials, monotone potentials and local potentials, the latter two being further generalized by Oyama and Tercieux [17] using iterated monotone potentials. Our condition is not a special case of the above sufficient conditions. It generalizes the ones in terms of zero-sum and best-response potential games. Our condition may guarantee the robustness of smaller sets of equilibria than the one in terms of generalized potential functions.

We also contribute to the literature on team vs. adversary games [7,25]. Von Stengel and Koller [25] show that team vs. adversary games have an equilibrium where team members receive the best payoff they can guarantee by independent randomization. We extend and generalize this result: in an infinite game with a saddle function there exists an equilibrium that induces an expectation of the saddle function equal to its maximin. We use the result to prove our sufficient condition for the robustness.

[1] For instance, Dziubinski and Goyal [6] and Clark and Konrad [5] study models with a single defender and a single attacker. Multiple defenders and a single attacker model is studied by Bier et al. [3]. Babaioff et al. [2] and Moscibroda et al. [16] study multiple defender/attacker models.

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