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A correspondence principle for cooperative differential equations ☆

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Abstract

This paper establishes a correspondence principle in a system of cooperative differential equations. Equilibria that are locally asymptotically stable must exhibit monotone comparative statics. In a wider context, all equilibria that exhibit monotone comparative statics are locally asymptotically stable. The orbits of such systems are nondecreasing in the parameter and the adjustment dynamics after a parameter change are monotone and convergent to a unique limit. Our results can be applied to such models as adaptive play in games with strategic complementarities, stochastic fictitious play in supermodular games, and the Matsuyama [6] model of global financial market liberalization with multiple capital goods.

JEL classification: C61; C62

Keywords: Correspondence principle; Monotone comparative statics; Asymptotic stability; Differential equations

1. Introduction

Consider a dynamic model where motion is described by a system of differential equations. The stationary states, which may be stable or non-stable, are the equilibrium points. If a stationary state is stable, it is a point of attraction in a small neighborhood around it. When the parameters

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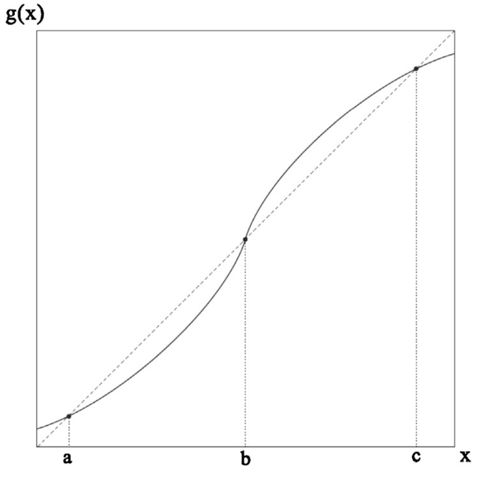

Fig. 1. An increasing function with three fixed points.

of the system are perturbed, the stationary states change. These dynamic movements are ignored in comparative static analyses, where attention is placed on exclusively a comparison of the stationary states before and after the perturbation. If an equilibrium increases when parameters are increased, then the equilibrium is said to have monotone comparative static property. More than half a century ago, Samuelson [11] pointed out that comparative static analysis should not be considered in isolation of the dynamic adjustments. The task that Samuelson set out for himself was to show that equilibrium points that exhibit monotone comparative statics were also stationary states that were stable, and vice versa. However, as pointed out in Arrow and Hahn [1], this so-called correspondence principle is true in the one-dimensional case only.

A simple example would illustrate the main ideas. Consider the following dynamical system: $\dot{x_1} = g(x_2) - x_1$ and $\dot{x_2} = g(x_1) - x_2$ in $[0, 1]^2 \subset \Re^2$ where $g : [0, 1] \to [0, 1]$ is increasing and continuous, as depicted in Fig. 1. Since g has three fixed points $\{a, b, c\} \subset [0, 1]$, the three stationary states are $\{(a, a), (b, b), (c, c)\} \subset [0, 1]^2$. Both (a, a) and (c, c) are stable stationary states. By contrast, (b, b) is a stationary state that is unstable. In terms of comparative statics, only (a, a) and (c, c) have monotone comparative statics property – if the mapping $g \times g$ is perturbed upwards (downwards), then the equilibrium point increases (decreases).[1] In this example, the correspondence principle holds exactly, because all stationary points have monotone static property and vice versa. A further observation is that, if the stationary state is stable and $g \times g$ is perturbed upwards (downwards), then the system moves from the old stationary state to a new stationary state of the same selection along an orbit that is monotone increasing (decreasing). By contrast, if the stationary state is unstable and $g \times g$ is perturbed upwards (downwards), then the system moves to a stationary state of a higher (lower) selection.

Recently, the correspondence principle has enjoyed a revival of interest, stemming from the seminal contribution of Echenique [3]. It begins with a correspondence in a game with strategic complementarities that maps from one period to the other in discrete time. This correspondence is assumed to be increasing in terms of the extreme points of the image set. The analysis con-

[1] Indeed, Milgrom–Roberts [8] Monotone Theorem implies the extreme fixed points have monotone comparative static property.