Optics and Lasers in Engineering 51 (2013) 311–316

Contents lists available at SciVerse ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng

Holographic particle tracking using Wigner–Ville distribution

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A R T I C L E I N F O

Article history:
Received 16 July 2012
Received in revised form
19 September 2012
Accepted 21 September 2012
Available online 16 October 2012

Keywords:
In-line particle holography
Particle tracking
Wigner–Ville distribution
Wigner distribution

A B S T R A C T

A new method for tracking particles from in-line holograms by using Wigner–Ville distribution is proposed. In comparison with the Wigner distribution, the proposed method has higher measurement accuracy. The experimental results show that the error in measurements of the axial position of the object depends mainly on the resolution of image sensor.

1. Introduction

In-line holography is an optical method for recording amplitude and phase of coherent light wave diffracted by a three-dimensional (3-D) object on photographic film [1,2]. One of its useful applications is to track position and measure size of small particles [3]. To record in-line particle holograms, opaque or semi-transparent particles are illuminated by a collimated coherent light. An interference between light waves diffracted from the particles and the directly transmitted light wave is recorded onto a photographic film and becomes hologram after wet-chemical development. The in-line particle hologram contains information about both 3-D spatial position and size of the particles which are encoded as a chirp signal and an envelope function, respectively. The frequency of the chirp signal is inversely proportional to the particle depth along the axial position z [3]. In a conventional analyzing method, this information is extracted by illuminating the developed hologram with the same coherent light. The transmitted light reconstructs the particle image at the same distance as its original position. Since, in practice, this distance is not known in advance, the image plane of best focus for each particle must be investigated by scanning the overall depth along an optical axis with fine steps. Although this method allows us to freeze moving particles and to analyze them later, we may deal with a huge number of particles in real applications. As a consequence, the conventional reconstruction process is very tedious and time consuming.

With development of electronic image sensors, the in-line holograms can be digitally recorded [4]. Several methods for extracting information from the digital holograms have been reported by solving the diffraction integral with a fast Fourier transform [5], Wigner distribution (WD) [6] and wavelet transform (WT) [7–10]. When the axial distance of the objects is unknown, the FFT-based method must reconstruct a set of images at different depths and search the in-focus image in order to obtain this distance. This method constitutes an iterative process, thus, it is also time consuming. On the other hand, the WT and the WD are methods based on a joint time–frequency representation of signals. Their abilities to determine simultaneously local frequency variations of non-stationary signals using particular analyzing windows can be employed to extract directly the depth information of the object from the in-line holograms.

In the WT, a bank of elementary windows is generated from a unique wavelet function which has a response of band-pass filter by dilations and translations [11–14]. A short analyzing window is employed at high frequencies, while long window is for low frequencies. This is consistent with a nature of non-stationary signals whose high-frequency components last in a relatively short time. By cross-correlating the input signal and the analyzing window, frequency contents of the two signals can be compared. A correlation output is produced when the signal has the same frequency content as the analyzing window. Due to the use of a bank of dilated windows, the WT is characterized by its multi-resolution property.

On the other hand, the WD employs the original signal itself as the analyzing window. The local frequency contents are extracted by using the auto-correlation operation, while its resolution is dependent upon the length of the signal being analyzed [15,16].

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http://dx.doi.org/10.1016/j.optlaseng.2012.09.009

In comparison with the WT, the computation of the WD is simpler and straight forward. It is a fact that when the WD is used to analyze real signals, its resultant output suffers from an inherent frequency artifact caused by interference between instantaneous positive and negative frequency components of the signals being analyzed [17]. This frequency artifact appears between the desired frequency contents, thus, the frequency localization may be affected. Since the in-line holograms record intensity of the interference patterns, their signals are purely real. This condition implies a corresponding negative frequency component for every positive frequency component. Consequently, the extraction of the axial distance of particle from the in-line holograms using the WD may be affected by the frequency artifact.

To obviate this artifact, the Wigner–Ville distribution (WVD) has been proposed for time- and space-frequency signal analyses [18–21]. The WVD employs analytic signals which comprise only positive frequency components to eliminate the frequency artifact. To our best knowledge, the WVD has never been used to analyze the in-line holograms. For these reasons, this work proposes a new method for improving tracking particles from the in-line hologram by using the WVD. The analytic hologram is numerically generated from the digital in-line holograms by using Hilbert function of Matlab. In the present work, the axial position of the particle can be accurately calculated from the local frequency variation obtained by using the WVD.

2. Theory

2.1. In-line particle holography

An amplitude transmittance of the in-line Fraunhofer hologram of a small spherical particle with a radius of a can be mathematically expressed as [3]

$$I(r) = 1 - \frac{2\pi a^2}{\lambda z}\sin\left(\frac{\pi r^2}{\lambda z}\right)\left[\frac{2J_1(2\pi ar/\lambda z)}{2\pi ar/\lambda z}\right] + \frac{\pi^2 a^4}{\lambda^2 z^2}\left[\frac{2J_1(2\pi ar/\lambda z)}{2\pi ar/\lambda z}\right]^2, \tag{1}$$

where λ and z are the wavelength of the illuminating light and the distance between the particle and the recording plane, respectively. J_1 denotes the first-order Bessel function, while r represents the radius coordinate in the hologram plane. The first term of Eq. (1) corresponds to the background which stems from light transmitted directly during the hologram recording. The second term represents the modulation of a sine chirp signal by an Airy function. The chirp signal encodes the recording distance z into its frequency content, while the Airy function corresponds to the particle size a. The third term is a square of the Airy function whose amplitude is much smaller than the other terms [22]. It is clear from Fig. 1 that the envelope of the chirp signal has a shape of the Airy function and the chirp frequency becomes higher as the spatial position increases.

2.2. The WVD

The WD and the WVD are mathematical techniques which have been introduced in signal analysis to overcome the inability of Fourier analysis to provide local frequency spectra. The WVD of an analytic signal $g_a(x)$ is defined as [18]

$$W_{g_a}(x,f) = \int_{-\infty}^{\infty} g_a(x+\xi/2)g_a^*(x-\xi/2)\exp(-i2\pi f\xi)d\xi, \tag{2}$$

where * stands for the complex conjugate. Eq. (2) can be considered as a Fourier transform of the instantaneous autocorrelation

$$R_{g_a}(x,\xi) = g_a(x+\xi/2)g_a^*(x-\xi/2), \tag{3}$$

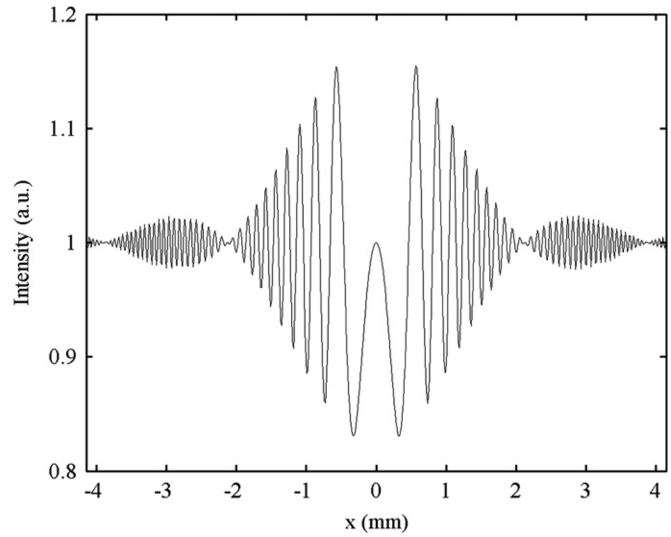

Fig. 1. Simulated in-line hologram of the spherical particle with the size $2a = 124.96$ µm and the axial distance $z = 40$ cm.

which is evaluated at position x for all ξ. The WD of a real signal $g(x)$ is defined as [15]

$$W_g(x,f) = \int_{-\infty}^{\infty} g(x+\xi/2)g^*(x-\xi/2)\exp(-i2\pi f\xi)d\xi, \tag{4}$$

which is similar to the definition of the WVD. Their difference can be understood by analyzing a sine chirp signal

$$g(x) = \begin{cases} \sin\left(\frac{\pi x^2}{\lambda z}\right) & \text{for} |x| \le L/2 \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

The analytic signal of this sine function can be mathematically expressed as

$$g_a(x) = \begin{cases} \exp\left[i\frac{\pi}{\lambda z}\left(x^2 - \frac{\lambda z}{2}\right)\right] & \text{for} |x| \le L/2 \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Substitution of Eq. (6) into Eq. (2) produces the WVD as

$$W_{g_a}(x,f) = 2(L-2|x|)\,\mathrm{sinc}\left[2(L-2|x|)\left(f - \frac{x}{\lambda z}\right)\right]. \tag{7}$$

Eq. (7) shows that the WVD output is a modulation of a sinc function which is centered at the instantaneous frequency $f = x/\lambda z$ by a triangle function whose peak appears at $x = L/2$. Therefore, the frequency content of the chirp signal concentrates in any position x around this instantaneous frequency. The sinc function appears as the result of the finite length of the signal duration. Consequently, the frequency resolution is dependent upon the signal duration. The plot of the WVD output corresponding to Eq. (7) along the positive direction x for $z = 40$ cm and $\lambda = 543.5$ nm is shown in Fig. 2(a). The vertical axis indicates the localized frequency variation f, while the horizontal axis is the spatial position x of the signal. The white color areas represent the correlation peaks with maximum values. It is apparent from this figure that the local frequency variation increases linearly with respect to the position. Its slope depends on the depth z of the object such that the deeper the depth, the milder the slope. Fig. 2(b)–(d) show the corresponding WVD outputs scanned along the spatial position x at the local frequency $f = 1$, 2 and 3 lp/mm, respectively. There are three correlation peaks that can be clearly identified from the figures. As the local frequency of the chirp signal becomes higher, the position of the peaks shifts accordingly