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On the relationship between conditional jump intensity and diffusive volatility☆

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ABSTRACT

In standard options pricing models that include jump components to capture large price changes, the conditional jump intensity is typically specified as an increasing function of the diffusive volatility. We conduct model-free estimation and tests of the relationship between jump intensity and diffusive volatility. Simulation analysis confirms that the tests have power to reject the null hypothesis of no relationship if data are generated with the relationship. Applying the method to a few stock indexes and individual stocks, however, we find little evidence that jump intensity positively depends on diffusive volatility as a general property of the jump intensity. The findings of the paper give impetus to improving the specification of jump dynamics in options pricing models.

1. Introduction

It is now standard to include jump components in models of underlying asset prices in order to evaluate options written on them. While jumps are rare, they have significant impacts on the welfare of investors. And precisely because they are rare, their properties are difficult to analyze. As a result, there is no consensus on how jumps should be modeled in terms of the conditional jump intensity and the conditional jump size. In this paper, we address the issue of modeling conditional jump intensity. More specifically, we focus on the relation between the conditional jump intensity and the diffusive volatility. In the development of options pricing models, diffusive volatility, also known as stochastic volatility, represents a major breakthrough after the Black–Scholes model of options pricing with a constant volatility. Stochastic volatility has been treated as the most important state variable, beside the price of the underlying asset, in determining options prices. It is thus very natural to specify the conditional jump intensity as an increasing function of the diffusive volatility of the underlying asset when jumps are added to the stochastic volatility models. In popular affine jump diffusion models, for example, the conditional jump intensity is specified as an affine function of the diffusive volatility of the underlying asset with positive slope coefficients (in non-trivial cases). It also seems natural to think that the probability of a jump is high when the diffusive volatility is also high. After all, both diffusive volatility and jump intensity are measures of the magnitude of possible future price changes.

In examining affine options pricing models, the relation between the conditional jump intensity and the diffusive volatility is estimated as part of the model. For example, Pan (2002) and Eraker (2004) find a significant increasing relation between the

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conditional jump intensity and the diffusive volatility of the underlying asset, using options data. Two issues involved with using options data make the results difficult to interpret. First, jumps in options prices can result from jumps in state variables, rather than from those in the underlying asset price. In potentially misspecified models, especially those that force the underlying asset price and other state variables to jump together, a relation between the jump intensity of the underlying asset price and diffusive volatility can be found spuriously. Second, since options pricing is conducted under the risk-neutral probability which involves risk premium, for models that implicitly assume no risk premium associated with jump intensity, a relation between the jump intensity and diffusive volatility under the risk-neutral probability may be attributed to that under the actual probability spuriously. Bates (2000) explicitly tests the difference in jump intensity-diffusive variance relation under the actual and risk-neutral probabilities and finds no relation under the former, but a positive relation under the latter. Using S&P 500 index return data only without options prices, Andersen et al. (2002) find an insignificant relation between the conditional jump intensity and the diffusive volatility. In short, the issues of whether there is a relation between jump intensity of the underlying asset price and its diffusive volatility, whether the relation is positive, and whether the relation is monotonic are unsettled.[1]

The results of parametric analysis of jump intensity can also be sensitive to the model assumptions. Recent studies have shown that many standard options pricing models are mis-specified. Jones (2003) finds that the square-root stochastic volatility model is incapable of generating realistic return behavior and the data are better represented by a stochastic volatility model in the constant-elasticity-of-variance class or a model with a time-varying leverage effect. Christoffersen et al. (2010) find that a stochastic volatility model with a linear diffusion term is more consistent with the data on the underlying asset and options than a stochastic volatility model with a square-root diffusion term is. Li and Zhang (2013) show that the affine drift of the diffusive volatility model is mis-specified because the mean reversion is particularly strong at the high end of volatility. These results suggest that the standard options pricing model with the square-root volatility process falls short of generating sharp increases and decreases in volatility when the level of volatility is high. As a result, when there are in fact spikes in the volatility, the model with a square-root volatility process attributes large changes in asset price to jumps and produces a spurious positive relation between the conditional jump intensity and diffusive volatility.

In this paper, we address the issue of whether the conditional jump intensity is an increasing function of the diffusive volatility under the actual probability. Since an exhaustive analysis of all possible combinations of specifications of jumps and diffusive volatility is impossible, we adopt nonparametric and semi-nonparametric approaches, which can reduce the chances of making erroneous inferences from mis-specification of parametric models.[2] We consider a simple method with many variations to identify jumps and choose among these variations through simulation to reduce the potential bias in examining the jump intensity-diffusive volatility relation, caused by the error in the estimated diffusive volatility. We then examine the relation between the conditional intensity of detected jumps and the estimated diffusive volatility for several stock indexes and individual stocks. Our results are based on intraday returns which enhance the powers of the jump detection and the test of jump intensity-diffusive volatility relation, compared with early studies based on daily returns. Using our more robust approaches, we arrive at the conclusion that the conditional jump intensity of most individual stocks and stock indexes we examine is unrelated to their diffusive volatility.

The finding that the conditional jump intensity is unrelated to the diffusive volatility for many indexes and individual stocks, contrary to what the standard jump-diffusion models assume, is important for understanding the dynamics governing underlying asset prices and deriving corresponding options pricing models. It prompts the study of what state variables really determine the conditional jump intensity and how to better model the relationship between these state variables and the jump intensity.[3]

The rest of the paper is organized as follows. Section 2 discusses the nonparametric jump detection methods. Section 3 conducts simulation analysis to examine the performance of the tests of the relation between conditional jump intensity and diffusive volatility. Section 4 presents the empirical analysis based on stock indexes and individual stocks. Section 5 concludes the paper.

2. The jump detection test and diffusive volatility estimators

Consider the log price of an asset, S_u, which follows a stochastic process,

$$dS_u = \mu_u du + \sqrt{v_u} dW_u + Z_u dN_u, \tag{1}$$

[1] Eraker (2004) also has estimation results without using options data in his Table 4 in which he does not report the relation between jump intensity and diffusive volatility and does not explain why not. Andersen et al. (2002) attribute the insignificance result to the approximation used to calculate standard errors, and a multicollinearity type problem. As shown by Andersen et al. (2002), when the conditional jump intensity is specified as an affine function of diffusive variance, instead of a constant, the standard errors of the parameters related to jump intensity are about 20 times larger.

[2] Nonparametric methods are applied to options pricing in Hutchinson et al. (1994), Aït-Sahalia (1996), Aït-Sahalia and Lo (1998), Broadie et al. (2000a,b), Aït-Sahalia et al. (2001), Aït-Sahalia and Duarte (2003), Li and Zhao (2009), and Li and Zhang (2010), among others.

[3] There is a separate line of research which models the conditional jump intensity as a function of realized past jumps and past conditional jump intensity, based on the observations that large price changes occur in clusters. Prominent works include Chan and Maheu (2002), Maheu and McCurdy (2004), Santa-Clara and Yan (2010), Yu (2004), Christoffersen et al. (2012), Maheu et al. (2013), and Aït-Sahalia et al. (2015). These studies address issues different from ours and use approaches different from ours.