Journal of Empirical Finance xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Journal of Empirical Finance

journal homepage: www.elsevier.com/locate/jempfin

Robust tests for a linear trend with an application to equity indices ☆

Sam Astill [a], David I. Harvey [b], Stephen J. Leybourne [b], A.M. Robert Taylor [c,*]

[a] Department of Economics, University of Warwick, UK
[b] Granger Centre for Time Series Econometrics and School of Economics, University of Nottingham, UK
[c] Essex Business School, University of Essex, UK

ARTICLE INFO

Article history:
Received 18 September 2013
Received in revised form 18 February 2014
Accepted 21 February 2014
Available online xxxx

JEL classification:
C22

Keywords:
Linear trend
Unit root tests
Strong serial correlation

ABSTRACT

In this paper we develop a testing procedure for the presence of a deterministic linear trend in a univariate time series which is robust to whether the series is I(0) or I(1) and requires no knowledge of the form of weak dependence present in the data. Our approach is motivated by the testing procedures of Vogelsang [1998, Econometrica, vol 66, p123–148] and Bunzel and Vogelsang [2005, Journal of Business and Economic Statistics, vol 23, p381–394], but utilises an auxiliary unit root test to switch between critical values in the exact I(1) and I(0) environments, rather than using this unit root test to scale the test statistic as is done in the aforementioned procedures. We show that our proposed tests have uniformly greater local asymptotic power than the tests of Vogelsang (1998) and Bunzel and Vogelsang (2005) when the error process is exact I(1), identical local asymptotic when the error process is I(0), and have better overall local asymptotic power when the error process is near I(1). Our proposed tests also display superior finite sample power to the tests of Vogelsang (1998) and Bunzel and Vogelsang (2005) and are competitive in finite samples with tests designed to be optimal in both the exact I(1) and I(0) environments. We apply our test procedures to a number of equity indices and find that these series appear to have a significant upward deterministic trend, yet are also highly persistent about this long run growth path.

1. Introduction

The ability to detect the presence and magnitude of a deterministic trend in an economic time series is of key importance when conducting empirical analysis, the presence of a linear trend being of particular relevance for the purpose of forecasting or testing for the presence of a unit root. In the latter case, failure to correctly specify a trend when it is indeed present is known to have an adverse effect, resulting in non-similar and inconsistent tests, as demonstrated by Perron (1998). Similarly, the power of unit root tests to reject the null under the I(0) alternative when a trend is unnecessarily included in a model specification is reduced, see inter alia Marsh (2009) and Elliot et al. (1996). The presence of a deterministic trend in an economic or financial time series can also be of interest in its own right, since a linear trend is compatible with a degree of underlying long run growth in the series. For example, this is of particular interest when considering the long run behaviour of stock prices and indices, where an underlying upward trend implies a long run average return. Moreover, the outcome of statistical tests of the efficient market hypothesis (EMH) is necessarily contingent on correct specification of the trend component of prices (or, equivalently, the mean component of returns).

☆ We thank the Guest Editors, Richard Baillie and Menelaos Karanasos, and two anonymous referees for their helpful comments on earlier versions of the paper.
* Corresponding author at: Essex Business School, University of Essex, Colchester, CO4 3SQ, UK.
E-mail address: rtaylor@essex.ac.uk (A.M.R. Taylor).

http://dx.doi.org/10.1016/j.jempfin.2014.02.004
0927-5398/

Testing for the presence of a linear trend is complicated by the fact that in practice it is typically not known whether the underlying process is I(0) or I(1). For example, uncertainty as to the degree to which financial markets are efficient suggests that one would not want to make an a priori assumption regarding the presence or absence of a unit root in the series from the outset. We therefore require tests for a linear trend that are robust to whether a series is I(0) or I(1) when determining whether a trend is present. There have been a number of papers suggesting testing procedures for detecting a deterministic trend function which are robust to the order of integration of the data including, inter alia, Vogelsang (1998), Bunzel and Vogelsang (2005), Harvey et al. (2007) and Perron and Yabu (2009). Vogelsang (1998) and Bunzel and Vogelsang (2005) employ an auxiliary unit root test statistic to line up the critical values of a t-statistic based on the levels of the data in the exact I(1) and I(0) environments. The approach taken in Harvey et al. (2007) utilises an auxiliary unit root or stationary test statistic to switch between the optimal trend function test in the exact I(1) and I(0) environments, and, as such, the test achieves the Gaussian asymptotic local power envelope in both cases. Perron and Yabu (2009) use a "super-efficient" estimate of the autoregressive parameter to construct a GLS based test statistic that also achieves the Gaussian asymptotic local power envelope in both the exact I(1) and I(0) cases.

Compared to the tests of Harvey et al. (2007) and Perron and Yabu (2009), the tests of Vogelsang (1998) and Bunzel and Vogelsang (2005) have the advantage of better size control in finite samples in the exact I(1) environment when the errors are i.i.d., albeit at the expense of relatively poor power properties, with the tests of Harvey et al. (2007) and Perron and Yabu (2009) fairly similar in terms of their overall performance. In the local to I(1) environment the results are less clear, with all tests displaying significant undersize, and no one test dominating in terms of overall power.

In this paper we propose a modification to the testing procedures of Vogelsang (1998) and Bunzel and Vogelsang (2005) in which an auxiliary unit root test statistic is used to scale the critical value of the test rather than the test statistic itself. We find the proposed modification yields a test that has uniformly greater local asymptotic power than the tests of Vogelsang (1998) and Bunzel and Vogelsang (2005) when the error process is exact I(1), has identical local asymptotic power when the error process is I(0) and has better overall finite sample properties. We also find that the proposed tests are competitive in finite samples when compared to the optimal tests of Harvey et al. (2007) and Perron and Yabu (2009).

The paper is organised as follows. Section 2 outlines the model. Extant tests for a deterministic linear trend are outlined in Section 3. In Section 4 we outline our proposed tests. In Section 5 the limiting distribution and local asymptotic power of the tests are detailed. Section 6 reports results of Monte Carlo simulations performed in order to assess the finite sample size and power properties of the proposed tests relative to existing tests. Section 7 reports results of an empirical exercise in which we apply the test statistics outlined in this paper to a number of US and UK equity indices. Concluding remarks are made in Section 8.

2. The linear trend model

Consider a sample of T observations generated according to the following data generating process (DGP)

$$y_t = \mu + \beta t + u_t, t = 1, \ldots, T \tag{1}$$

$$u_t = \alpha_T u_{t-1} + \varepsilon_t, t = 2, \ldots, T. \tag{2}$$

Following Vogelsang (1998), Bunzel and Vogelsang (2005) and Harvey et al. (2007) we assume that the process $\{\varepsilon_t\}$ is such that

$$\varepsilon_t = C(L)e_t, C(L) := \sum_{i=0}^{\infty} C_i L^i$$

with $C(z) \neq 0$ for all $|z| \leq 1$ and $\sum_{i=0}^{\infty} i|C_i| < \infty$, and where $\{e_t, \mathcal{F}_t\}$ is a martingale difference sequence with $E(e_t^2|\mathcal{F}_{t-1}) = 1$ and $sup_t E(e_t^4|\mathcal{F}_{t-1}) < \infty$. We also define $\omega_\varepsilon^2 := lim_{T\to\infty} T^{-1} E(\sum_{t=1}^{T} \varepsilon_t)^2 = C(1)^2$. The initial condition, u_1, is assumed to be $O_p(1)$. These assumptions ensure that we can apply a Functional Central Limit Theorem (FCLT) to the partial sums of $\{\varepsilon_t\}$, so that $T^{-1/2}\sum_{t=1}^{\lfloor rT \rfloor} \varepsilon_t \xrightarrow{d} \omega_\varepsilon w(r)$ where $\lfloor . \rfloor$ denotes the integer part of its argument, $\xrightarrow{d}$ denotes weak convergence and $w(r)$ is a standard Wiener process.

The autoregressive parameter in Equation (2) determines the order of integration of the series. When $\alpha_T = \alpha$ and $|\alpha| < 1$ the series is I(0), whereas if $\alpha_T = 1 - \overline{\alpha}/T$ the series is near I(1), with $\overline{\alpha} = 0$ corresponding to an exact I(1) process. Under these assumptions a FCLT applies to the partial sum of $\{u_t\}$ defined as $S_t := \sum_{j=1}^{t} u_j$. When $\{u_t\}$ is I(0), $T^{-1/2} S_{\lfloor rT \rfloor} \xrightarrow{d} \sigma w(r)$, where $\sigma^2 := C(1)^2/(1-\alpha)^2$. When $\{u_t\}$ is near I(1), $T^{-1/2} u_{\lfloor rT \rfloor} \xrightarrow{d} \omega_\varepsilon w_{\overline{\alpha}}(r)$, where $w_{\overline{\alpha}}(r) := \int_0^r exp(-\overline{\alpha}(r-s))dw(s)$.

The null hypothesis of interest is $H_0 : \beta = \beta_0$. This null hypothesis can be tested against either the two-sided alternative $H_1 : \beta \neq \beta_0$, or against either the right-tailed alternative $H_1' : \beta > \beta_0$ or the left tailed alternative $H_1'' : \beta < \beta_0$. The leading case of interest is where $\beta_0 = 0$, so that the null and alternative hypotheses signify the absence or presence of a linear trend, respectively. When analysing the asymptotic performance of the tests outlined in this paper it will prove useful to consider the local alternative hypotheses of $H_{10} : \beta = \beta_0 + \kappa T^{-3/2}$ and $H_{11} : \beta = \beta_0 + \kappa T^{-1/2}$, where κ is a finite constant, with the scalings $T^{-3/2}$ and $T^{-1/2}$ providing the appropriate Pitman drifts under I(0) and I(1) errors, respectively.

Please cite this article as: Astill, S., et al., Robust tests for a linear trend with an application to equity indices, J. Empir. Finance (2014), http://dx.doi.org/10.1016/j.jempfin.2014.02.004