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Counter-cyclical risk aversion

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ABSTRACT

The paper proposes a consistent estimator of time-varying risk aversion in consumption-based CAPM. Based on the Epstein–Zin–Weil (Epstein and Zin, 1989, 1991; Weil, 1989) recursive utility, we derive the Euler equation in which risk aversion is a non-parametric function of time. The proxy variable method is utilized to replace the unobserved return to aggregate wealth in the Euler equation. The estimation of risk aversion is carried out based on a two-stage local-linear regression method. Given the estimate, we investigate the conventional wisdom in economics that risk aversion is counter-cyclical. The empirical results strongly support the counter-cyclicality of the risk aversion parameter.

1. Introduction

The conventional wisdom in economics and finance is that risk aversion is counter-cyclical. Investors are willing to invest in risky assets during economic booms, and conversely they prefer to invest in relatively risk-free assets during recessions. In other words, investor's risk aversion is believed to move against business cycles. While this belief seems to appeal to both practitioners and academia, the majority of leading asset pricing models cannot explore the plausible counter-cyclicality of risk aversion, because most of such models assume that risk aversion is a parameter *constant over time*. Moreover, there has been little effort in the literature to verify whether this idea of counter-cyclical risk aversion is *empirically* supported. Given this limitation on asset pricing models and the lack of empirical evidence in literature, we propose a model with time-varying risk aversion, and estimate the parameter to verify whether this conventional wisdom is indeed supported by data.

Although it has been of rather limited use and its empirical aspects have not been studied thoroughly, the notion of time-varying risk aversion itself has been in the literature. In the area of consumption-based capital asset pricing model (CCAPM), Campbell and Cochrane (1999) and Cechetti et al. (2000) suggest that asset markets can be interpreted via the channels of fluctuating risk aversion or distorted beliefs. Brandt and Wang (2003) formulate a CCAPM in which aggregate risk aversion is time-varying in response to news about consumption growth and inflation. Schwert (1989), Beltratti and Morana (2006) and Morana (in press) consider stock market volatility as a proxy for time-varying risk aversion, measured by means of a GARCH model or through a time series model using

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realized volatility. Despite these various implications, there has been little effort to verify through formal estimation whether risk aversion is indeed counter-cyclical. This paper undertakes the task.

In order to investigate its cyclical behavior, we first need to model time-varying risk aversion in an appropriate utility function. To this end, we borrow the framework of the Epstein and Zin (1989, 1991) and Weil (1989). It is well-known that the Epstein–Zin–Weil recursive utility provides us with a convenient separation of risk aversion and elasticity of intertemporal substitution (EIS). We replace the traditional fixed risk aversion by a nonparametric function in continuous time, and derive the corresponding Euler equation. The only assumption on the risk aversion parameter function is that it changes smoothly over time. That is, we do not need any parametric assumptions on risk aversion.

The estimation of the parameter function is conducted by a *two-stage* local linear regression. We first divide our sample into numerous overlapping blocks of equal length. We propose this block method to ensure that risk aversion is "asymptotically constant within each block". As long as the parameter is fixed within each block, the risk aversion can be estimated block-wise by the generalized method of moments (GMM) (Hansen, 1982). We call this first-stage estimation method a block-wise GMM. In the second stage, we apply the local linear regression (Cleveland, 1979) to these block-wise GMM estimates to smooth out the estimates. This two-stage estimation of risk aversion provides us with a *consistent* estimate of the parameter function in time, as shown by Theorem 1 in Section A.2.

It is absolutely important to realize that the second-stage local-linear regression on the block-wise GMM estimates is essential to the consistent estimation of time-varying risk aversion. Traditional GMM works *only if* the parameter is constant. That is, for time-varying parameters, this method cannot be applied in general. However, under the assumption that risk aversion is constant within a very small block (see Remark 1 of Section A.2), GMM can be used as part of the overall estimation procedure. This assumption requires infinitely many blocks (see Remark 2 of Section A.2), which eventually leads to the estimation of infinitely many block-wise parameters. That is, we need to estimate a parameter function, which explains why the block-wise GMM alone cannot provide a consistent estimator of risk aversion. Unless we are willing to assume piece-wise constant risk aversion, which is a very restrictive assumption, we need to work under the general assumption of smoothly-varying risk aversion. Hence the second-stage local smoothing on the block-wise GMM estimates is essential to ensure the consistency of our estimator.

In general, the estimation of time-varying parameters in the literature has been based on pre-specified parametric data-generating processes (DGP), such as a random walk and an autoregressive moving average (ARMA) processes. For example, Brandt and Wang (2003) assume a stationary AR(1) process for risk aversion. This approach is, however, highly vulnerable to the problem of possible model *mis-specification*. A sample of works on the mis-specification problem includes Fan and Gijbels (1996). The use of a non-parametric function for a time-varying parameter can be considered as a solution to this potential problem. In other words, the non-parametric local-linear regression on the block-wise GMM estimates can reduce the possibility of mis-specification.

The main contribution of this paper is that we provide an asymptotic justification to our two-stage estimation of time-varying risk aversion. Although the block-based method is a natural approach to consider in estimating time-varying parameters embedded in conditional moments, there has been no asymptotic justification of this approach under the assumption that the parameter changes smoothly in time. In our framework, the risk aversion parameter is a continuous-time function with no pre-specified DGP on it. A minimum smoothness condition on the risk aversion ensures the consistency of our two-stage local-linear estimator. In proving the consistency of the proposed estimator, the main difficulty lies in that the introduction of a non-parametric risk aversion function leads to both dependent and non-stationary model errors. To overcome this difficulty, we let the errors be *locally stationary* (Kim, accepted for publication; Kim et al., 2010), which is a mild form of non-stationarity (see Remark 5 of Section A.2), and apply the methodology recently developed by Kim et al. (2010).

The second contribution is that we present empirical evidence, based on formal estimation, that risk aversion is *counter-cyclical*. We compare our estimated risk aversion with a measure of the U.S. business cycle, namely, the U.S. unemployment rate. There turns out to be a close co-movement between the two series (see Figs. 1 and 2). The correlation between risk aversion and the unemployment rate is around 0.3. The majority of the previous work on the counter-cyclicality of risk aversion is based on simulation studies involving parametric DGPs. In this study, we only introduce a smoothness condition on the risk aversion parameter, and carry out its estimation based on empirical data. To the best of our knowledge, this is the first time that risk aversion in the Epstein–Zin–Weil recursive utility is shown to be counter-cyclical through the consistent estimation of the parameter function.

The organization of this paper is as follows: Section 2 introduces the CCAPM based on the Epstein–Zin–Weil recursive utility function with time-varying risk aversion. The dynamics for consumption and dividend growth rates and the framework for the risk aversion parameter function are also defined. Section 3 describes the consumption and return data used to estimate the Euler equation. Section 4 discusses how to construct a two-stage local linear estimator based on the block-wise GMM estimates. Section 5 discusses empirical results. Economic interpretation of the results is also provided. Section 6 concludes the paper and discusses future research. The derivation of the Euler equations and the asymptotic properties of our two-stage estimator are provided in Appendix A.

2. CCAPM with time-varying risk aversion (TV-RA)

2.1. Epstein–Zin–Weil with TV-RA

Since the seminal works by Breeden (1979) and Grossman and Shiller (1981), the consumption-based capital asset pricing model (CCAPM) has been extensively used in modern asset pricing literature. In particular, the recursive utility function plays an important role in the CCAPM area due to its nice properties that the traditional power utility function does not possess. Originally suggested by

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