Journal of Empirical Finance 23 (2013) 33–47

Contents lists available at ScienceDirect

Journal of Empirical Finance

journal homepage: www.elsevier.com/locate/jempfin

Value at risk forecasts by extreme value models in a conditional duration framework

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ARTICLE INFO

Article history:
Received 16 June 2011
Received in revised form 3 May 2013
Accepted 6 May 2013
Available online 14 May 2013

JEL classification:
C22
C58
F30

Keywords:
Extreme value theory
Autoregressive conditional duration
Value at risk
Self-exciting point process
Conditional intensity

ABSTRACT

The analysis of extremes in financial return series is often based on the assumption of independent and identically distributed observations. However, stylized facts such as clustered extremes and serial dependence typically violate the assumption of independence. This has been the main motivation to propose an approach that is able to overcome these difficulties by considering the time between extreme events as a stochastic process. One of the advantages of the method consists in its capability to capture the short-term behavior of extremes without involving an arbitrary stochastic volatility model or a prefiltration of the data, which would certainly affect the estimate. We make use of the proposed model to obtain an improved estimate for the value at risk (VaR). The model is then compared to various competing approaches such as Engle and Marianelli's CAViaR and the GARCH-EVT model. Finally, we present a comparative empirical illustration with transaction data from Bayer AG, a typical blue chip stock from the German stock market index DAX, the DAX index itself and a hypothetical portfolio of international equity indexes already used by other authors.

1. Introduction

The recent financial crisis has prompted several approaches of varying sophistication to estimate more properly the VaR and other risk measures. However, financial institutions have shown that most of these approaches performed poorly, which is mainly because extreme events tend to cluster during periods of market turbulence. This clustering behavior is therefore a critical issue for practitioners since a series of VaR failures during a period turmoil could lead to insolvency.

McNeil and Frey (2000) consider this cluster behavior and address the conditional quantile problem by proposing a method for applying extreme value theory (EVT) to the conditional return distribution through a two-stage method that combines GARCH models for forecasting volatility and EVT techniques applied to the residuals from the GARCH analysis. This is called the GARCH-EVT approach. Although this methodology works quite well in practice, it has a major drawback: there exists no theory of the extreme clustering behavior based on the residuals of a GARCH model, a problem which is addressed by Mikosch (2003).

A novel way to deal with the cluster of extremes is to use a marked self-exciting point process version of the Peaks Over Threshold (POT) approach, introduced preliminarily in Chavez-Demoulin et al. (2005) as the Hawkes–POT model. The main characteristic of this approach is that the intensity of the occurrence of extreme events is allowed to depend on past extreme events as well as the size of the exceedances, thus allowing for more realistic models. In this paper we pursue a similar idea, but from the point of view of the dynamics given by inter-exceedance times among extreme events. The main objective of this paper

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http://dx.doi.org/10.1016/j.jempfin.2013.05.002

is therefore to investigate the short-term behavior of extremes without involving an arbitrary stochastic volatility process that would certainly impact the risk measures. To this end, the method proposed in this paper concentrates only on the dynamics given by the time of occurrence of the extreme events and the exceedances above a high threshold previously defined, and how these can influence the future behavior of the process.

The contribution of this paper is a model that is able to capture this cluster behavior. Accordingly it is called autoregressive conditional duration peaks over threshold model (ACD-POT). Since the introduction of the first ACD model by Engle and Russell (1998) a plethora of modifications and alternatives has been proposed for modeling different characteristics in irregularly spaced time series. For instance, Ghysels and Jasiak (1998) proposed modeling the volatility of irregularly spaced data by means of an ACD-GARCH model, while Russell and Engle (2005) propose the autoregressive conditional multinomial (ACM) model for the conditional distribution of discrete price changes. For this reason, it seems natural to model the inter-exceedance times for cluster of extreme events by means of an ACD model while the sizes of exceedances are still modeled by generalized Pareto distributions, according to the POT approach.

One of the advantages of the ACD-POT approach over the classical GARCH-EVT approach is the explicit modeling of the conditional intensity for the inter-exceedance times, which can be viewed as a proxy for conditional volatility of exceedances.[1] Another advantage of the ACD-POT approach is that the VaR estimates exhibit both monotonically decreasing and increasing shapes. This contrasts the Hawkes–POT model, whose self-exciting functions are restricted to monotone decreasing functions. Therefore, the shape of an ACD-POT model has an interesting interpretation in periods of financial turmoil, where the VaR estimates typically increase in an initial period, then become close to constant before finally decreasing.

The ACD-POT model is applied to a set of transaction data from financial markets. In particular, we provide cross-sectional evidence for a single stock market index, the Bayer AG, the DAX index, which itself is a market value-weighted portfolio, and a portfolio of international equity indexes analyzed previously in McNeil et al. (2005) and Chavez-Demoulin et al. (in press). Empirical results show that characteristics associated with previous extreme losses, as well as the time between these extreme events, have a significant impact on the dynamic aspects and size of future extreme events. In addition, the results of our backtesting procedure show that the models proposed are suitable for estimating different risk measures such as the VaR, in accordance with the restrictions imposed by the Basel Committee on Banking Supervision (1996, 2006). Further, in comparison to other competitive models, the ACD-POT models generally outperform the basic specifications of the CAViaR models introduced by Engle and Manganelli (2004). Finally, the ACD-POT and the two-stage GARCH-EVT method (McNeil and Frey, 2000) are the only ones that eradicate the threat of VaR violation clustering in the analyzed returns.

This paper is organized as follows. In Section 2, we outline relevant aspects of the classical POT model and we give a brief motivation for modeling the inter-exceedance times between extreme events. In addition, we describe the linking of the ACD model to the POT method, which results in the ACD-POT model theory that is central to the paper. In Section 3, we present a classification scheme and introduce the framework for selecting the model that is best suited to return modeling and the VaR requirements. Moreover, we present two alternative strategies for benchmarking the ACD-POT method. The models are then applied to transaction data from Bayer AG, the DAX index and a hypothetical portfolio. Conclusions and proposals for future work are discussed in Section 4.

2. Methodology

In this section we develop the methodology of the new approach. We start with a brief review of some basic concepts of EVT.

2.1. Classical extreme value theory

In the context of financial time series particular interest lies often in studying the losses arising from extreme events in a composite stock market index (e.g. DAX, Dow Jones, Nasdaq) or a stock index of an individual company (e.g. Microsoft, IBM or Facebook, among many others). Extreme losses may be regarded as returns whose size (in absolute terms) is larger than a sufficiently high threshold $u > 0$. Additionally we are interested in the times of occurrences of these losses t_i with $0 < t_1 < \ldots < t_i < \ldots$ and their magnitudes y_i.

Actually, we consider a set of observations $\{(t_i, y_i)\}_{i \geq 1}$ belonging to a two-dimensional space $\mathscr{Y} = (0,1) \times (u,\infty)$, where the time has been rescaled for convenience to the interval (0,1). The history of this process can be understood as $\mathscr{H}_t = \{(t_i, y_i)\ \forall i: t_i < t\}$, which consists only of the occurrence times and marks up to time t but not including t. We display this idea in Fig. 1. The time event t_i is the time of the i-th peak exceedance, and we refer to this process as the ground process. $y_i - u$ is the magnitude of the exceedances for a sufficiently high threshold u and we will call this process the process of marks (in Fig. 1 this refers to observations $t = 2, 6, 8, \ldots$ or equivalently $i = 1, 2, 3, \ldots$).

Let $N(t) \equiv N((0,t) \times y)$ represent the marked point process (MPP) that characterizes this stochastic process. Moreover, we define a ground point process $N_g(t) \equiv N_g((0,t))$, which refers to the stochastic process of the inter-exceedance times (i.e. the time between extreme events).

[1] See Engle and Russell (1998) Section 8.1 about the relationship between the price intensity and volatility.