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Implied liquidity: Model sensitivity

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ABSTRACT

The concept of implied liquidity originates from the conic finance theory and more precisely from the law of two prices where market participants buy from the market at the ask price and sell to the market at the lower bid price. The implied liquidity λ of any financial instrument is determined such that both model prices fit as well as possible the bid and ask market quotes. It reflects the liquidity of the financial instrument: the lower the λ, the higher the liquidity. The aim of this paper is to study the evolution of the implied liquidity pre- and post-crisis under a wide range of models and to study implied liquidity time series which could give an insight for future stochastic liquidity modeling. In particular, we perform a maximum likelihood estimation of the CIR, Vasicek and CEV mean-reverting processes applied to liquidity and volatility time series. The results show that implied liquidity is far less persistent than implied volatility as the liquidity process reverts much faster to its long-run mean. Moreover, a comparison of the parameter estimates between the pre- and post-credit crisis periods indicates that liquidity tends to decrease and increase for long and short term options, respectively, during troubled periods.

1. Introduction

The theory of conic finance extends the classical finance theory by dropping the law of one price in favor of the law of two prices. In the classical one-price model the market acts as counterparty to investors and accepts any amount of financial assets which are traded at the going market price whatever the direction of the trade. On the other hand, the conic finance theory recognizes the existence of two market prices for financial assets: market participants buy from the market at the ask price but sell to the market at the lower bid price. The difference between the ask and the bid price often referred to as the bid–ask spread is an indication of the market liquidity.

The conic finance theory originates from the framework of acceptability proposed by Cherny and Madan (2009) in which risk measures are defined in terms of distorted expectations of zero cost cash-flows X. More precisely, we say that a risk X is acceptable if

$$\mathbb{E}_Q[X] \geq 0 \text{ for all measures } Q \text{ in a convex set } M.$$

The convex set M contains the supporting measures, which can be seen as a kind of test measures under which the cash-flow needs to have positive expectation to deliver acceptability. Under a larger set M, one has a smaller set of acceptable risks, because there are more tests to be passed. Cones of acceptability were defined by Cherny and Madan (2009) and depend solely on the distribution

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function $F_X(x)$ of X and on a distortion function Ψ: X is acceptable if the distorted expectation is non-negative. More precisely, the distorted expectation of a random variable X with distribution function $F_X(x)$ relative to the distortion function Ψ is defined as

$$\int_{-\infty}^{+\infty} x d\Psi(F_X(x)). \tag{1.1}$$

Such obtained risk-adjusted distribution functions typically allocate more weight to the down-side (losses) than to the up-side (profits) than the original distribution function. In this paper, we will typically consider the MINMAXVAR distortion function:

$$\Psi_\lambda(y) = 1-\left(1-y^{\frac{1}{\lambda+1}}\right)^{\lambda+1}, \quad \lambda\in\mathbb{R}_+, y\in[0,1],$$

where the acceptability index λ quantifies the degree of distortion.

In the aftermath of the recent credit crunch, market participants and regulators have shown an increasing interest in the assessment of liquidity risk which typically dries up during market turmoil periods. The current standard consists of considering the bid–ask spread of financial instruments as an indicator of their liquidity. Nevertheless, it is a well known fact that the bid–ask spread can change in a non-linear way with the volatility and the spot price, without the financial product's liquidity changing. The law of two prices provides an alternative tool to assess liquidity by introducing the concept of implied liquidity which isolates and quantifies the liquidity risk in a more fundamental way. This concept was proposed by Corcuera et al. (2012) and illustrated under the Black–Scholes setting.

The concept of implied liquidity is defined as follows for the exemplary case of a call option. The two-price model is calibrated by first inferring the implied volatility σ from the mid call price and by then determining the liquidity parameter λ in order to replicate as well as possible the market bid and ask quotes (or equivalently the bid–ask spread) while keeping σ fixed. The as such obtained implied liquidity reflects the liquidity of the call option: the lower the λ, the higher the liquidity. This paper assesses the model sensitivity of implied liquidity. Besides the traditional way of determining an implied volatility under the Black–Scholes model, alternative implied volatility settings have been proposed. More specifically, Lévy and Sato based implied volatility concepts have been worked out in (Corcuera et al., 2009) and (Guillaume, 2011). Considering more flexible distributions allows the reduction of the skew adjustment which is inherent under the Black–Scholes model in equity markets. In particular, Corcuera et al. (2009) have shown that implied Lévy space volatility models are able to translate the typical Black–Scholes volatility skew into a flat Lévy implied volatility curve (i.e., an implied volatility independent of the option strike) and outperform, in this regard, implied Lévy time volatility models. From the definition of acceptability (Eq. (1.1)), it is clear that the derivation of the implied liquidity of any derivative contract requires to know the distribution function of the underlying asset in closed-form. This turns out to not be the case for most of the option pricing models proposed in the financial literature. A major exception consists of the Black–Scholes model. However, in Section 4, we will provide a semi-closed-form approximation for the bid and ask prices under the Variance Gamma models by expanding the cumulative distribution function of the call payoff as a weighted sum of Black–Scholes call payoff distribution functions (see Guillaume, 2011; Madan et al., 2013). We will thus be able to extend the implied liquidity concept under these Lévy/Sato VG space volatility settings in a straightforward way.

Moreover, this paper contains a study of the evolution of the implied liquidity pre- and post-crisis, giving an indication of the liquidity risk during market turmoil periods and providing an implied liquidity time series. The analysis of these time series can be seen as a first step towards stochastic liquidity modeling.

2. The implied volatility models

This section recalls the different models under investigation: the Black–Scholes model, the implied Lévy volatility models (see Corcuera et al., 2009) and the implied Sato volatility models (see Guillaume, 2011). The implied Lévy and Sato volatility models are built on the widespread concept of implied volatility, but on the basis of more flexible distributions than the Normal distribution. It is indeed well known that the Normal distribution does not capture very well neither historical stock returns nor stock price distributions inferred from option prices. Market returns in reality are characterized by both skewness and excess of kurtosis, ubiquitous features present in both stock time series and option price surfaces.

2.1. The Black–Scholes model

Although the shortcomings of the Black–Scholes model have been highlighted by many authors (see for instance Cont, 2001 or Schoutens, 2003), it has remained the standard tool to quote vanilla options in terms of the implied volatility. This is most likely due to its relatively simple concept and by the intuition that traders have developed in this model parameter.

Under the Black–Scholes model, the stock price process is modeled by a geometric Brownian motion:

$$S_t = S_0 \exp\left(\left(r-q-\sigma^2/2\right)t + \sigma W_t\right), \quad t\geq 0,$$

where $\sigma > 0$ is called the volatility parameter, r denotes the risk-free rate, q the dividend yield and $W = \{W_t, t \geq 0\}$ is a standard Brownian motion.