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How certain are we about the certainty-equivalent long term social discount rate?

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ABSTRACT

Theoretical arguments for using a term structure of social discount rates (SDR) that declines with the time horizon have influenced government guidelines in the US and Europe. The certainty equivalent discount rate that often underpins this guidance embodies uncertainty in the primitives of the SDR, such as growth. For distant time horizons the probability distributions of these primitives are ambiguous and the certainty equivalent itself is uncertain. Yet, if a limited set of characteristics of the unknown probability distributions can be agreed upon, 'sharp' upper and lower bounds can be defined for the certainty-equivalent SDR. Unfortunately, even with considerable agreement on these features, these bounds are widely spread for horizons beyond 75 years. So while estimates of the present value of intergenerational impacts, including the social cost of carbon, can be bounded in the presence of this ambiguity, they typically remain so imprecise as to provide little practical guidance.

Introduction

The outcome of cost-benefit analysis of public projects with intergenerational consequences is notoriously sensitive to the social discount rate (SDR) employed. Small variations in assumptions about the appropriate SDR can therefore lead to very different policy recommendations for the preservation of natural resources and environmental quality, including the retention of biodiversity (Freeman and Groom, 2013) and the case for mitigating against greenhouse gas emissions (e.g. Nordhaus, 2007; Stern, 2008).

This policy-sensitivity is particularly problematic because the primitives that underlie the long-term discount rate are difficult to determine. For example, the growth rate of aggregate consumption and the rate of return to capital over the next four centuries are essentially unknown today, since they depend on a number of unpredictable events including technological advances, political and social unrest, environmental change and even pandemics (e.g. Almond, 2006).

A typical way to approach long-term discounting is to calculate a 'certainty equivalent' social discount rate, a single rate which embodies uncertainty in the SDR primitives. Yet even though uncertainty is taken into account, such calculations assume a fanciful level of predictive power, since they assume perfect knowledge of the relevant probability distributions. In the context of intergenerational decision-making, the probabilities associated with different future states of the world are

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thought to be ambiguous at best, and at worst unknown.[2] Consequently, the certainty equivalent discount rate is itself uncertain.

In this paper we make a contribution to the literature on social discounting under uncertainty by calculating empirical 'sharp' upper and lower bounds for the certainty-equivalent social discount rate when we have imperfect knowledge of probability distributions of SDR primitives. Such bounds can be calculated if decision-makers are willing to assume partial, but not complete, agreement on some characteristics of these distributions. The existence of sharp bounds is the good news. The bad news is that these bounds are typically very wide and fail to provide precise calculations of present values.

These findings are important because the burgeoning literature on the term structure of social discount rates, expertly reviewed by Gollier (2012) and Arrow et al. (2014), has been highly influential at a policy level. The message coming from these contributions is that, for risk free projects, the term structure should be declining with the time horizon. This view is exemplified by a recent *Policy Forum* article in *Science*, in which it is argued that where we are uncertain about the future 'there are compelling arguments for using a declining discount rate schedule' (Arrow et al., 2013, p. 350). As a consequence of these theoretical advances, declining discount rates (DDRs) can now be found in government guidelines in the UK and France, influence recommendations in the US (Cropper et al., 2014), and lie behind recent advice given to the Norwegian, Danish and Dutch governments. In the UK, DDRs have been used in the governmental economic analysis of the High Speed 2 (HS2) rail link and for capital budgeting purposes by the Nuclear Decommissioning Authority. DDRs have already had policy impact.

The DDRs that appear in government guidelines are typically based on certainty-equivalent discount rates which reflect uncertainty in the future or disagreement among experts on the appropriate discount rate, perhaps for ethical reasons. An influential set of arguments supposes that for some x, the different definitions of which are reviewed in subsequent sections, the present value, p_H, of a certain \$1 arriving at time H is given by $p_H = E[\exp(-Hx)]$. The H-period certainty-equivalent discount rate, R_H, is then defined through the relationship $\exp(-HR_H) = p_H$. Exponential functions are convex, and so, by Jensen's inequality, $E[\exp(-Hx)] \geq \exp(-HE[x])$: uncertainty over x raises the present value, p_H, and lowers the discount rate, R_H. The magnitude of this effect becomes greater the more uncertain we are about x , and the more convex the exponential function, the latter being determined by the parameter value H. As a consequence R_H declines with the time horizon until in the limit, as $H \to \infty$, it approaches the lowest possible outcome for x.[3]

While this may seem like a narrowly defined structure, it has several different interpretations depending on the approach taken to social discounting and DDRs. Its most famous use stems from Weitzman's (2001) 'Gamma Discounting' paper. Here, x was interpreted as reflecting different expert opinions on the value of the discount rate itself. In this context, the justification for using the formula $p_H = E[\exp(-Hx)]$ remains controversial. This is partly because its connection with utility theory was not made clear at the time, and partly that more recent theoretical motivations rely on quite restrictive assumptions. We discuss this point in detail in DDRs under gamma discounting and The theoretical basis for DDRs sections. Less well known is that x can also be interpreted through the social rate of time preference (SRTP) in a more standard consumption-based Ramsey asset pricing framework. A proof of this proposition is given in The theoretical basis for DDRs section. A third interpretation of x, also discussed in The theoretical basis for DDRs section, is that it represents the average return to risk-free capital over the horizon of the cash flow. Since they all have a similar expectations structure, the methods that we describe for deriving the sharp bounds for the certainty equivalent discount rate can be equally well applied to any of these interpretations.

Putting any of these interpretations into practice requires assumptions about the uncertainty surrounding the primitives of the SDR that are contained in x, through its probability density function (pdf), $f(x)$. The main approach taken so far is to parameterize $f(x)$ and treat this distribution as if it is perfectly identified. Yet, because our knowledge of the future is nowhere near as precise as this approach would suggest, a more realistic starting point would be to admit that we do not, perhaps cannot, know the true nature of $f(x)$ over time horizons of many decades or centuries. For very long-term decisions the context is one of uncertainty and ambiguity. We are not alone in thinking this. Pindyck (2015) recently made a similar point in relation to social discounting, while Iverson (2013) and Traeger (2014) both take ambiguity as their starting points. Yet the approach that we take assumes far more knowledge about the future than Knight (1921), who would maintain that true uncertainty is immeasurable. To reduce the problem we imagine a situation where the social planner gathers a panel of economists who, while accepting that it will be impossible to agree on the precise distribution of x, are nonetheless tasked with identifying the *set* of density functions such that all members agree that $f(x)$ is a member of this set.[4] Agreement in this context takes the form of agreeing characteristics shared by all distributions within a set. For instance, the set may contain probability distributions which share the same first K moments, or alternatively the same values for particular quantiles.[5] They might, instead, be members of a family of distributions, such as the

[2] We use the term 'ambiguous' in this paper in the sense of Klibanoff et al. (2011, p. 400) 'that this definition is characterized by, roughly, disagreement in the probability assigned to an event by the various probability measures that are subjectively relevant.'

[3] A numerical example illustrates the mechanics of the result. Suppose that, with equal probability, x will either take the value of 2% or 6%. The social value of \$1 delivered at time H with certainty is then given by the expected present value under these two outcomes, $p_H = 0.5(\exp(-0.02H) + \exp(-0.06H))$, resulting in $R_1 = 3.98\%$, $R_{50} = 3.13\%$, $R_{100} = 2.67\%$ and $R_{400} = 2.17\%$. The $x = 6\%$ outcome is, through the power of exponential discounting, given increasingly less voice in the social valuation p_H as H gets larger. For horizons of a century or more, to good approximation, its contribution to p_H becomes so small that it can be ignored altogether with $p_H \approx 0.5\exp(-0.02H)$ and $R_H \approx 0.02 - \ln(0.5)/H$. In the limit, as $H \longrightarrow \infty$, $R_H \to 2\%$

[4] While it is possible that these assumptions will be falsified with the benefit of hindsight, we assume that the social planner is willing to make decisions on the basis of assumptions about $f(x)$ that are sufficiently uncontroversial for reasonable people to be able to agree upon them today. O'Hagan et al. (2006) provides a detailed review of how experts' probability judgements might be assessed for this purpose.

[5] To avoid issues around infinities, as famously discussed in a related context through the 'dismal theorem' of Weitzman (2009), we assume throughout that the first K moments of $f(x)$ are finite and that, more generally, its moment generating function is defined.