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Equivalent energized coil model for magnetic field of permanent-magnet spherical actuators

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ABSTRACT

Magnetic field modeling is extremely important for electromagnetic (EM) driven spherical actuators. This paper proposes a novel mathematic modeling method based on equivalent energized coil and Biot–Savart law to formulate the complex magnetic field distribution in three-dimensional (3D) space. The energized coil model is employed as an equivalent substitution of cylindrical permanent magnet (PM) poles. The magnetic field distribution of single energized coil is then formulated analytically. The complete magnetic field model of the actuator with multiple cylindrical PM poles is thus achieved from linear superpositions. Compared with other conventional approaches, as there are no omission of high order of harmonic terms, shape approximation of magnet poles and assumption of evenly distributed flux field, it helps to improve the modeling accuracy. Furthermore, this method is more generic for other flux field applications. It is available for both PM and EM poles, and theoretically could be implemented for other magnet shapes. The computational time may increase for complex magnet shapes and distribution patterns. The proposed method is applied to the spherical actuator with novel 3D magnetic pole array that helps to improve the actuator torque output. Numerical computation is conducted to validate the derived analytical magnetic field model. It shows that the analytical model fits with the result from finite element method (FEM) closely. A research prototype and an automatic experimental platform have been developed. Experiment is thus conducted to measure the magnetic field distribution of the spherical actuator. The data comparison shows that the analytical model matches the experimental measurement result well. The developed model can be employed for subsequent study of torque formulation and control implementation.

1. Introduction

Spherical actuator is a device that can achieve multiple degree-of-freedom (DOF) rotary motions in a single joint. Due to its compact structure, smooth motion and fast response in contrast to the present multi-DOF mechanical devices with several single-axis motors [1,2], it has potential applications in robotics, manufacturing, micro/nano technology and electric vehicles [3,4]. Various designs based on different operating principles have been proposed by researchers [5–7]. Among them, spherical actuators driven by electromagnetic (EM) forces have been widely studied [8–17]. Magnetic field modeling is extremely important for these electromagnetic actuators, because it significantly influences the torque generation and control implementation [18–20]. Researchers have proposed modeling approaches to formulate the complex magnetic distribution of spherical actuators with different structures. There are three typical approaches, i.e., equivalent magnetic circuit method (EMC), analytical solution to quasi-Poissonian/Laplace's equations and distributed multipole method (DMP). The EMC method is usually used to formulate the magnetic field of single-axis electrical machine with narrow air gaps [21]. Li et al. have extended it to model the three-dimensional (3D) magnetic field of multiple degree-of-freedom (DOF) spherical actuators [22]. Lumped magnetic circuits or reluctance networks are used to represent the magnetic flux path in electrical machines [23,24]. It generally assumes that the magnetic flux is distributed evenly in the air gap without leakage, which unavoidably compromises the accuracy of analytical results. The modeling method

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by utilizing analytical solution to quasi-Poisssonian/Laplace's equations is widely employed to formulate the magnetic field of PM spherical actuators. For example, Davey et al. analyzed the 3D distribution of the magnetic field of spherical actuator by using magnetic field vector potential [25]. Yan et al. systematically analyzed the magnetic field of a spherical actuator with multiple dihedral-shaped PM poles from Laplace's and Poisson's equations [26]. Wang et al. formulated 3D magnetic field of 2/3-DOF spherical actuators based on Laplace's equation and magnetic scalar potential [27,28]. Spherical-shaped PM poles magnetized radially and in alternative directions are employed for the designs. When PM poles of other shapes such as common cylinders are utilized, geometric approximation is necessary, which in turn may reduce the accuracy of the analytical formulation significantly. Meanwhile, the resolving process of harmonic series is relatively complex. To simply the computation, the fundamental terms of spherical harmonics are usually utilized for the magnetic field formulation, which may compromise the modeling precision further. The DMP method is proposed by Lee and Son et al. to solve the magnetic field of spherical wheel motors [29,30]. It can be used to formulate magnetic field of permanent magnet poles and helps to obtain analytical models in closed form. However, the accumulation of a number of dipoles may cause accuracy loss in a certain degree.

The objective of this paper is to propose a novel equivalent energized coil model to formulate the complex magnetic field in 3D space of electromagnetic spherical actuators. It deals with the cylindrical PM poles as an equivalent energized coil, and then obtains the analytical expression of magnetic field distribution based on Biot–Savart law conveniently. Compared with conventional approaches, as there are no assumptions on evenly distributed flux field in air gap, omission of high order harmonics and shape approximation, it helps to improve the modeling precision of magnetic field in space. Furthermore, this method is more generic for modeling applications of other electromagnetic actuators. It is available for both PM and EM poles, and theoretically not constrained by pole geometry. It could be implemented to actuators with different pole shapes and patterns, but the computational time may increase much due to complexity of magnets. Numerical computation is employed to validate the derived analytical model. Following that, a research prototype of spherical actuator with novel 3D magnet array [31,32] is developed for experimental purpose. An automatic multi-DOF experimental apparatus is designed and fabricated to measure the magnetic flux distribution in the actuator. The experimental data is then utilized for comparison with the derived analytical models.

The rest of the paper is organized as follows. Section 2 introduces the schematic structure and operating principle of the proposed spherical actuator. Section 3 focuses on the analytical formulation of the magnetic field. Numerical computation is conducted in Section 4 and the result is compared with the analytical model. The research prototype and experimental works are presented in Section 5. The study is concluded in Section 6.

2. Schematic structure and operating principle

The schematic structure of the spherical actuator is illustrated in Fig. 1. The actuator consists of one rotor and one stator assembled together through the ball bearing that helps to support the rotor and ensure the concentricity of stator and rotor. Coils are mounted on the stator surface in the pattern of sphere. More coils can be incorporated in longitudinal direction to improve the torque output and working range. Fig. 1(a) and (b) present the system assembly view and the relationship between stator and rotor, respectively. Fig. 1(c) shows that the rotor consists of two spherical-shell-like surfaces on which the PM poles are distributed symmetrically. The two layers of spherical surfaces are jointed together with four L-shaped connecting pieces. In other words, the PM poles are no longer constrained on a single 2D spherical surface in conventional designs. They are extended to 3D space including the radial direction. The similar pattern can be implemented into the stator poles. The 3D poles pattern helps to incorporate more magnets or coils into the system, improve the magnetic flux density and thus increase the torque output of the spherical actuators. To reduce the system cost, cylindrical PM poles are employed for the actuator design.

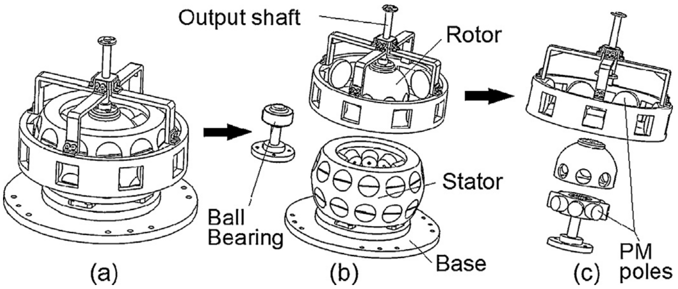

Fig. 1. Schematic structure of the spherical actuator (a) Assembly view; (b) Exploded view of stator and rotor; (c) Rotor poles distribution.

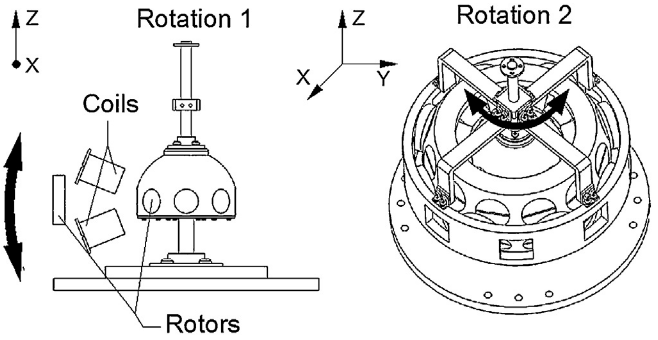

Fig. 2. Three degree-of-freedom motions of the spherical motor.

The PM poles mounted on the rotor surfaces generates magnetic field in the 3D actuator space. The magnetic field interacts with the current inputs in the stator coils, and produces torque vectors in the space to move the rotor in 3-DOF. One set of coils can be energized to create tilting motions of rotor in x direction, and another set in orthogonal direction to create the second tilting motions in y direction. Rotation 1 in Fig. 2 represents the two similar tilting motions. The rest of the coils can be used to create spinning motions about the rotor axis, i.e., Rotation 2 illustrated in Fig. 2.

3. Analytical formulation of magnetic field

3.1. *Magnetic field modeling of single PM pole*

3.1.1. *Magnetic vector potential and flux density*

Magnetic field of the spherical actuator is essentially a vector field in 3D space. According to Helmholtz's Theorem, to formulate a vector field, we may utilize the divergence and curl of the field, i.e., $\nabla \cdot \vec{B}$ and $\nabla \times \vec{B}$, where $\vec{B}$ is the vector of magnetic flux density. It is known that the steady magnetic field is a source free field, i.e.,

$$\nabla \cdot \vec{B} = 0. \tag{1}$$

Meanwhile, there must be a magnetic vector potential field $\vec{A}$ that can satisfy the equation of $\nabla \cdot (\nabla \times \vec{A}) = 0$. Hence, the magnetic flux intensity $\vec{B}$ can be expressed as

$$\vec{B} = \nabla \times \vec{A}. \tag{2}$$