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Bistable behavior of electrostatically actuated initially curved micro plate

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ABSTRACT

The axisymmetric snap-through of an initially curved circular micro plate, subjected to a distributed electrostatic force is studied. The analysis is based on a reduced order (RO) model resulting from the Galerkin decomposition, with buckling modes of a flat plate used as the base functions. The results of the RO model are compared with the results available in the literature for initially flat plate and with numerical results obtained for both cases of displacement-independent "mechanical" load and displacement-dependent electrostatic load. The study indicates that a model with at least three degrees of freedom (DOF) is required for an accurate prediction of the equilibrium path, under both types of loading. The analysis shows that due to the nonlinearity of the electrostatic load, the snap-through occurs at a lower displacement than under the "mechanical" load. The presented results also indicate that micro plates of realistic dimensions can be actuated by reasonably low voltages, suggesting the feasibility of the usage of such elements for various applications.

1. Introduction

Many nonlinear systems are distinguished by multistability, namely an ability to be found in multiple (stable) states at the same values of governing parameters. Bistable structural elements, like beams, plates and shells, exhibit two stable configurations, under the same loading. The transition between the two is commonly referred as snap-through buckling. The behavior of structures liable to snap-through, due to deformation-independent "mechanical" (and thermal) loads, is a well established topic in structural mechanics [1] which continues to attract attention of researchers [2].

Traditionally, the buckling phenomenon was considered as a failure which should be avoided. However, progress in the design of devices where snap-through behavior could be beneficial [3] stimulated renewed interest in the mechanics of bistable beams [4,5] and shells [6,7]. In the realm of micro- and nanoelectromechanical systems (MEMS/NEMS), applications of these elements include switches [8], sensors [9], non-volatile memories [10] and micropumps [11–16], just to name a few. These devices are often actuated by nonlinear, configuration dependent, electrostatic forces, inducing instabilities such as the electrostatic pull-in. Static and dynamic snap-through and pull-in behavior were extensively investigated by various analytical, numerical and experimental approaches for the case of one-dimensional (1D) bistable structures, mainly curved beams (e.g., see [17–20] and references therein).

Recently, electrostatically actuated 2D structures, such as membranes or plates, drew increasing attention, thus resulting in works aiming at understanding their behavior in various loading conditions. Relatively large body of works was devoted to the pull-in behavior of electrostatically actuated membranes (e.g., see [21] and references therein). In [22–24], the pull-in and natural frequencies of circular and rectangular microplates were analyzed using RO models. The same approach was used in [25,26] to study the effects of Casimir forces and thermal stresses. These effects were considered also in [27], where a step-by-step linearization combined with finite differences analysis was used, and in [28] where a direct numerical shooting method was implemented. Approximate interrelationship between residual stress, critical voltage, and pull-in deflection of an axisymmetric circular plate was derived in [29]. The asymmetric pull-in of circular and annular plates under combined electrostatic and thermal actuation was analyzed in [30]. An investigation of the static and dynamic behavior of a fully clamped rectangular flat microplate was carried out in [31] using RO and FE models, based on the von-Kármán plate model. The static and dynamic cylindrical bending of a plate clamped at the two opposite edges was studied numerically and experimentally in [32]. To account for the imperfections, which are liable to appear during

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fabrication, a small curvature was introduced to the model. However, the presence of this small initial curvature, which is also inclined towards the electrode, did not evoke bistability under static loading.

While all these works considered electrostatically actuated initially flat plates, the behavior of initially curved plates under nonlinear, configuration depended, and specifically electrostatic forces, was not considered so far. Flat plates are not bistable and exhibit only the pull-in collapse [32]. In contrast, as we show in the present work, 2D bistable structures such as initially curved plates, subjected to electrostatic load, are liable to both snap-through and pull-in. The nonlinear static behavior of such systems is the subject of the present research. Note that the symmetric and asymmetric snap-through of bistable spherical cups, made of magnetic polymeric material, and loaded by generally nonlinear, deflection dependent, magnetic force was studied experimentally in [33]. However, due to large distance between the loading magnet and the structure, the influence of nonlinearity of the magnetic loading was not pronounced. In addition, RO modeling aspects were not addressed.

The goal of the present work is twofold. First, we develop a compact RO model of the electrostatically actuated curved plate and investigate its reliability, accuracy and suitability for the description of this kind of structures. Second, we demonstrate, using the model, the actuation feasibility of a micro plate with realistic dimensions in a bistable regime using reasonably low voltages.

2. Formulation

Consider a shallow, initially curved, circular micro plate, shown in Fig. 1. The plate has thickness $\hat{d}$, which is small compared to the radius R. The plate is assumed to be made of homogeneous isotropic linearly elastic material with Young's modulus E, and Poisson's ratio ν. The initial, as-designed, shape of the stress-free plate is described by the function $\hat{w}_0\left(\hat{r},\theta\right)=\hat{h}_0 z_0(\hat{r},\theta)$, where $\hat{h}_0$ is the elevation of the plate central point above its edge, and $z_0\left(\hat{r},\theta\right)$ is a non-dimensional function such that $\max_{\hat{r}\in[0,R]}\left[z_0\left(\hat{r},\theta\right)\right]=1$. The plate is actuated by transverse electrostatic distributed load generated by a planar electrode which is located at a distance $\hat{g}_0$ from the plate circumferential boundary.

On the basis of the Kirchoff hypothesis, combined with the non linear von-Kármán strain–displacements relations, the axisymmetric equilibrium equations of a clamped circular curved plate actuated by transverse electrostatic was developed using the Hamilton's principle, resulting in the following two equations

$$\frac{d}{d\hat{r}}\left(\hat{r}\hat{N}_{\hat{r}\hat{r}}\right)-\hat{N}_{\theta\theta}=0 \tag{1}$$

$$-\frac{1}{\hat{r}}\left(\frac{d^2}{d\hat{r}^2}\left(\hat{r}\hat{M}_{\hat{r}\hat{r}}\right)\frac{d\hat{M}_{\theta\theta}}{d\hat{r}}\right)=\frac{1}{\hat{r}}\frac{d}{d\hat{r}}\left(\hat{r}\hat{N}_{\hat{r}\hat{r}}\frac{d\hat{w}}{d\hat{r}}\right)-\frac{\epsilon_0 V^2}{2\left(g_0+\hat{w}\right)^2} \tag{2}$$

with the following boundary conditions

$$\hat{u}=\hat{w}=0,\quad \frac{d\hat{w}}{d\hat{r}}=0 \quad @ \quad \hat{r}=R \tag{3}$$

$$\hat{u}=0,\quad \frac{d\hat{w}}{d\hat{r}}=0 \quad @ \quad \hat{r}=0 \tag{4}$$

with $\hat{w}$ being finite at the plate center, where $\hat{u}$ represents the radial displacement [34]. In addition, $\epsilon_0=8.854\times10^{-12}$ F/m is the permittivity of the free space and V is the voltage difference between the plate and the electrode.

Table 1
Non-dimensional quantities used in the development.

$r\triangleq\hat{r}/R$	Radial coordinate
$u\triangleq\hat{u}/R$	Radial displacement
$w\triangleq\hat{w}/g_0,\quad w_0\triangleq\hat{w}_0/g_0$	Plate config./initial config.
$h_0\triangleq\hat{h}_0/g_0$	Initial elevation at the center
$d\triangleq\hat{d}/g_0$	Thickness
$\alpha\triangleq g_0/R$	Parameter
$\gamma\triangleq\left(R^2E\hat{d}\right)/\left(D\left(1-\nu^2\right)\right)$	Membrane load parameter
$N_{ij}\triangleq\left(R^2/D\right)\hat{N}_{ij}$	In-plane tensile and shear loads
$M_{ij}\triangleq\left(R^2/(Dg_0)\right)\hat{M}_{ij}$	Bending and torsion
$\beta\triangleq\left(\epsilon_0V^2R^4\right)/\left(2g_0^3D\right)$	Voltage parameter
$\beta^M\triangleq qR^4/(g_0D)$	"Mechanical" load parameter

The membrane forces and the moments given in Eqs. (1)–(3) can be expressed in terms of the plate mid-surface displacements

$$\hat{N}_{\hat{r}\hat{r}}=\frac{E\hat{d}}{1-\nu^2}\left(\frac{d\hat{u}}{d\hat{r}}+\frac{1}{2}\left(\left(\frac{d\hat{w}}{d\hat{r}}\right)^2-\left(\frac{d\hat{w}_0}{d\hat{r}}\right)^2\right)+\nu\frac{\hat{u}}{\hat{r}}\right) \tag{5}$$

$$\hat{N}_{\theta\theta}=\frac{E\hat{d}}{1-\nu^2}\left(\nu\left(\frac{d\hat{u}}{d\hat{r}}+\frac{1}{2}\left(\left(\frac{d\hat{w}}{d\hat{r}}\right)^2-\left(\frac{d\hat{w}_0}{d\hat{r}}\right)^2\right)\right)+\frac{\hat{u}}{\hat{r}}\right) \tag{6}$$

$$\hat{M}_{\hat{r}\hat{r}}=-D\left(\frac{d^2\left(\hat{w}-\hat{w}_0\right)}{d\hat{r}^2}+\frac{\nu}{\hat{r}}\frac{d\left(\hat{w}-\hat{w}_0\right)}{d\hat{r}}\right) \tag{7}$$

$$\hat{M}_{\theta\theta}=-D\left(\frac{1}{\hat{r}}\frac{d\left(\hat{w}-\hat{w}_0\right)}{d\hat{r}}+\nu\frac{d^2\left(\hat{w}-\hat{w}_0\right)}{d\hat{r}^2}\right) \tag{8}$$

where $D\triangleq E\hat{d}^3/\left(12\left(1-\nu^2\right)\right)$ is the flexural rigidity.

An analytical solution of Eqs. (1)–(8) is available only for initially flat plates ($w_0=0$), for the special linear case (without the stretching-bending coupling) under "mechanical" loads [34]. Hence, a RO model is used here. This approach is commonly used for analysis in MEMS due to its compact form and relative simplicity when compared to the finite elements (FE) analysis, which can be computationally costly. In addition, since available commercial FE software packages do not allow seamless analysis of electrostatically loaded structures, combined with continuation methods for tracking of unstable branches [35], development of custom-built codes is usually required. On the other hand, RO models are very suitable for the design-stage modeling, parameters evaluation and feasibility studies, as well as system level multi-physics analyses. The development of a compact, yet reliable RO model, is one of the goals of this work.

2.1. Reduced order model

In order to analyze the plate response, a reduced order model based on the Galerkin decomposition is constructed. For convenience, a non-dimensional representation is used according to Table 1. The deformed shape of the plate is approximated by the series (the summation convention is valid)

$$w(r)\approx q_i\Phi_i(r)\qquad u(r)\approx p_iH_i(r) \tag{9}$$

where $i=1,\ \ldots,\ n$ is the mode/DOF number, and q_i, p_i are the generalized coordinates of the elevation and radial displacements, respectively. The base functions $\Phi_i(r)$ and $H_i(r)$

$$\Phi_i(r)=C_i\left(J_0(\lambda_ir)-J_0(\lambda_i)\right)\qquad H_i(r)=P_iJ_1(\lambda_ir) \tag{10}$$

are the eigenfunctions (buckling modes) associated with the linear homogeneous counterpart of Eq. (2) with $V=0$ and $N_{rr}=\text{const}$, which describes a flat circular plate, made of a homogeneous