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Geometric quasi-similarity: Case of nozzles with quadrant-shaped inlet

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ABSTRACT

Geometric similarity (i.e. difference only in size scale) is generally believed to be unavoidable condition without which it is impossible to apply the Buckingham π−theorem to two mutually related flow-fields. Recently introduced idea of secondary invariants can by-pass this limitation and accept geometric quasi-similarity—i.e. cases with different values of ratios of geometric parameters. In this paper this new approach is demonstrated on an example case of a single-parameter family of nozzles which are mutually not fully geometrically similar.

1. Introduction

The ultimate target of research in physical sciences is discovery of ***invariants*** of the investigated problem. Once discovered, invariants are the key to formulation of governing laws. In fluid mechanics, the invariants are usually dimensionless complexes set up from problem variables. The primary invariant for a particular problem, Fig. 1, is the numerical value of basic complex.

It is generally known that similarity – the powerful tool for approaches to solution of problems in fluid mechanics – admits only size scale. Only this makes the invariants found in one case transferable to another flowfield. If there is, however, only a limited number of geometric parameters that are different in investigated cases, it is possible to extrapolate the basic ideas of similarity and evaluate the corresponding secondary invariants. These then make possible full description of both geometrically quasi-similar cases. This resembles the basic idea of the local similarity applied successfully in [1] to the problem of analytic solution of turbulent jet. Instead of a single universally valid similarity solution for the whole jet, the quasi-similarity leads to continuous series of locally valid results, in the jet case gradually varying with increasing axial distance from the nozzle.

In this paper, this idea is extended to geometric quasi-similarity. It is demonstrated on fluid flows inside nozzles. This is a case of flows in which the question of invariants was until recently complicated by the fact that the used characterisation parameter – discharge coefficient (or the related Euler number) – is strictly speaking not really invariant. Only recently the true primary invariant for nozzles, shape parameter c_T, was derived in [2]. Method of identifying secondary invariants discussed in the present article is based on measurements performed on family of laboratory nozzle models. Parameters c_T for individual nozzles formed together the dependence, analogous to Fig. 2. This was used to derive the secondary invariants. While a primary invariant characterises by a single value a particular family member i.e., particular nozzle, the family as a whole is characterised by two secondary invariants. The obvious advantage gained by the secondary invariants is the possibility of solving such tasks like optimum nozzle geometry for a particular application.

The methodology of identifying the secondary invariants thus consisted of three steps:

1) Setting up a mathematical model based on the hypotheses formulated in [2] and identifying the nozzle shape parameter c_T as the primary invariant.
2) Experimental work: measurement of hydraulic losses for incompressible, low Mach number flows over a very wide range of Reynolds numbers and verification of the primary invariance.
3) Then the relative lengths l/d of nozzle exit channels formed the parameter of the family. Indentification of the secondary invariants for the geometric quasi-similar family of nozzles by analysis of the dependence $c_T = f(l/d)$.

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Nomenclature

A	Coefficient of power-law fit (–)
a	Secondary invariants (–)
b	Secondary invariants (–)
Bo	Boussinesq number (–)
c_T	Invariant coefficient (–)
d	Nozzle exit diameter (m)
Eu	Euler number (–)
e	Specific energy (J/kg)
Δe	Drop in fluid specific energy (J/kg)
l	Length of constant-diameter exit channel (m)
$\dot{M}$	Mass flow rate (kg/s)
Ha	Hagenbach correction term (–)
Ha_0	Hagenbach term for fully developed flow (–)
P	Pressure (Pa)
ΔP	Pressure drop (Pa)
Q	Quadratic dissipance (m^2/kg^2)
Qt	Loss-less dissipance (m^2/kg^2)
Re	Reynolds number (–)
r	Radius of quadrant-shaped inlet (m)
Te	Dimensionless parameter for convenient presentation of the universal law (–)
$\dot{V}$	Volume flow rate (m^3/s)
v	Fluid specific volume (m^3/kg)

Greek alphabet letters

δ_*	Dsplacement thickness of boundary layer (m)
ν	Fluid viscosity (m^2/s)

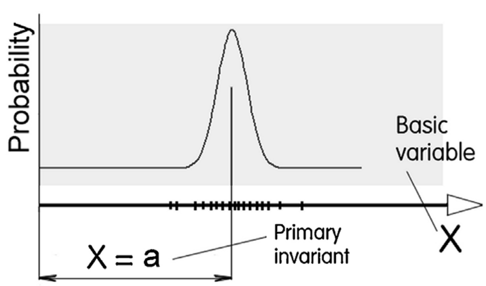

Fig. 1. Schematic representation of identifying a primary invariant from experimental or computed data. The invariant is the value of the basic variable X at which the data attain their highest probability.

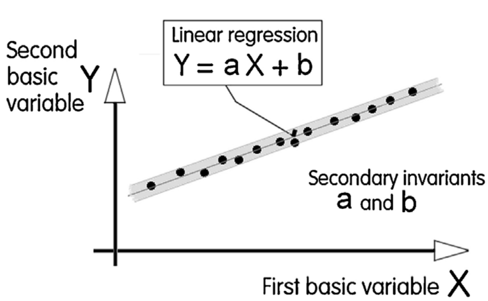

Fig. 2. Meaning of the secondary invariants: they are the constants of dependence on the primary invariant of a family of studied mutually not exactly similar cases. The invariants are preferably defined by the parameters of least-squares linear fit to the suitable transformed dependence.

2. Nozzles

Fluid flow supplied into a nozzle leaves there the closed conduits and issues as a jet. Practical importance of nozzles is due to extensive use of jets in many engineering tasks like cooling, drying, or heating by jet impingement [3]. Jets are also used in agitating suspensions, in burners, jet pumps, and more recently also in no-moving-part fluidic jet-type devices like amplifiers [4,5] and oscillators [6,7]. Especially the oscillators found recently popularity in intensification of various processes, typically reactions in chemical microreactors.

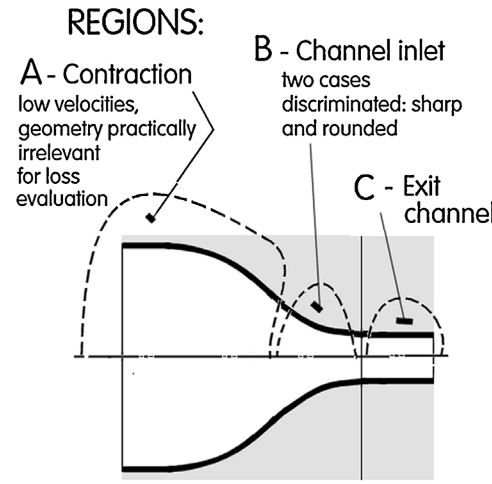

Fig. 3. Components of a typical nozzle. Most nozzle designs differ in the shape of their contraction part A—which, due to the low flow velocity prevailing there, has very little effect on the overall nozzle pressure loss behaviour.

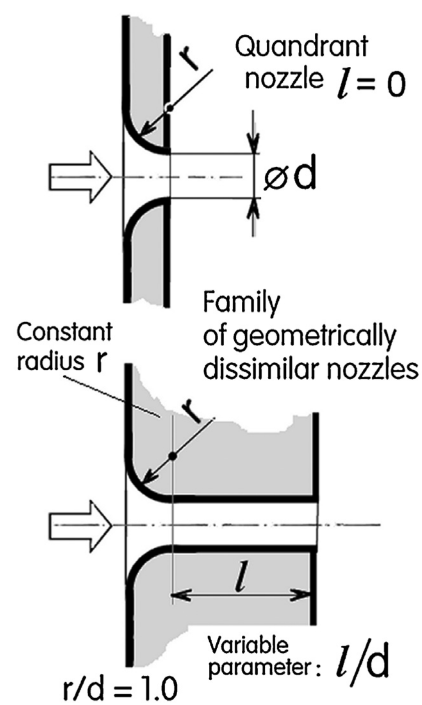

Fig. 4. The particular case of nozzles discussed in this paper as example of a family with determined secondary invariants. It is a single-parameter family with the same rounding radius r = d of entrance into exit channels having different lengths *l*. The quadrant nozzle (top), known from its use as low-Re flowmetering orifice [10,15,28,29], may be thought of as the extreme zero-length case.

Nozzles used in this paper to demonstrate the search for secondary invariants were studied in [2]. They are of axisymmetric shape as presented in Fig. 4, operated at very low Mach numbers, i.e. in flows not influenced by compressibility. Important components of these nozzles as presented in Fig. 3 are their exit channels. Mem-