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Large market games, the law of one price, and market structure☆

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ABSTRACT

This paper introduces a new class of market games featuring multiple posts per commodity, in which trading posts are privately owned. It is demonstrated via three robust counterexamples, that in this setting the law of one price fails, thus showing, contrary to longstanding belief in the literature, that price dispersion in *large* market games is extremely robust. Most importantly, it is established that even in economies with a continuum of small agents and infinitely many atoms (all of whom can arbitrage prices if they so wish), and an infinite number of markets *per commodity*, the set of equilibria—and the resulting market structure—is influenced, both by strategic behaviour, *and* private ownership of posts.

1. Introduction

The influence of strategic considerations on the process of price formation is a fundamental issue, and one elegant theory capturing both these concepts is that of *strategic market games* (SMG). An SMG, originating in Dubey and Shubik (1978), Shapley and Shubik (1977), and Shubik (1973), is a noncooperative trading model in which commodity prices depend on the buy-and-sell decisions of agents. Such strategic decision making by agents has called into question the validity of the *law of one price*[1] (LOP) in SMG with multiple "trading posts" per commodity (MTPC). An important implication of the failure of the LOP is that the set of equilibria depends nontrivially on the structure of trading posts. Regarding the latter, it is interesting to note that in standard SMG models, trading posts are typically assumed to be publicly and costlessly available. Surprisingly, the question never seems to have been asked if (or how) the "privatisation" of trading posts would affect the equilibrium allocations and prices in a meaningful manner. The purpose of this paper is to demonstrate that private ownership of trading posts,[2] alongside strategic behaviour by agents, in MTPC market games is indeed a material issue.

The following fact is established: even in *very large* economies, strategic considerations *still matter* in the determination of the equilibrium market structure. We achieve this by proving, in a new class of MTPC market games, that the LOP, an intimate feature of Walrasian markets, fails to obtain in very general settings. Hence, trading posts cannot be consolidated.[3] This is in stark contrast to the norm in SMG, where provided there are at least countably infinitely many agents (large and/or small), the LOP always prevails. Indeed, Koutsougeras (1999, 2003a, b) shows the failure of the LOP when the number of agents is finite. Koutsougeras (2003a) then proves that as the number of agents increases without bounds in MTPC models with publicly-available posts, the uniformity of prices across trading posts is restored, independently of the characteristics—preferences, endowments, measure, etc.—of agents.[4] In our model, however, the failure of the LOP to obtain effectively stems from the heterogeneity of agents. There are two

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1 The LOP postulates that *at equilibrium*, there is a single price that clears all markets for a commodity. It is a *central feature* of Walrasian economies, in which markets for a commodity are consolidated and modelled as a single trading spot where transactions take place. See Koutsougeras (1999, 2003b) for numerical examples in which the LOP fails in SMG with finitely many agents, all of whom face no *binding* liquidity constraints.

2 It must be noted that in the model that we propose, post owners are not given any kind of "extreme" market power. More precisely, they can neither "close down" their post, nor preclude agents from trading at their spots, such that all agents are perfectly free to choose where to trade, and to arbitrage prices should they so wish.

3 The equilibrium market/trading-post structure is determined by the distribution of prices across posts for each commodity. Consider any commodity k. If for k the support of this distribution is a single point, then there is effectively a single trading post for k. However, if equilibrium prices are not uniform across trading posts for k, then it follows that the equilibrium structure of posts cannot be merged into a single trading platform. So, the LOP—or the failure thereof—is a "tool" that we use to determine the market structure at equilibrium.

4 Koutsougeras' (2003a) limit economy need not be atomless—even if there exist *finitely* many *non-price-taking* atoms in the limit, the LOP *must still hold*. Thus, note that that limit economy need not be perfectly competitive; indeed, the validity of the LOP is a more general issue than the prevalence of perfect competition.

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types of agents in this model, pure traders (the only kind of agents that Koutsougeras, 1999, 2003a, b, considers), and trading post owners. The latter not only buy and sell across posts, but also levy a proportional "service charge" on agents who trade at their spots. We show, intriguingly, that even with a continuum of price takers, and only as few as two, or as many as an infinite number of "large" players, the LOP is still violated. This persistent price inequality is driven, both by strategic play by agents, and the trading-post service charge, an intricate concept which looks deceptively trivial.

To the best of our knowledge, none of the existing SMG models analyses the existence and availability of trading posts. Thus far, the SMG literature has been content to assume that trading posts somehow exist, and are somehow made publicly and costlessly available for all agents to trade at, as though by *some other* "invisible hand". In this paper, we depart from the literature and introduce post owners, who in addition to commodities, are endowed with trading posts. These agents also engage in trade, by presenting arrays of buy-and-sell strategies at their own, and/or other post owners' posts, as do the pure traders. Now, in most—if not all—economic models, privately-owned trading platforms are very rarely provided free of charge, and are even more so in real-world economies. In this light, we assume that post owners levy a *proportional* service charge per unit of (monetary) *net* trade on all agents who transact at their post.[5] Thus, an agent whose net trade is zero at a post—and therefore derives no direct monetary benefit from trading there—has no premium to pay. This service charge is reminiscent of the taxes and transactions costs that agents pay and incur in Gabszewicz and Grazzini (1999), Koutsougeras and Ziros (2015), and Rogawski and Shubik (1986). However, differently to these models, in the current paper, it is *agents who charge agents*,[6] and solely on their *net proceeds*, a formulation which is unique to this SMG. We assume that these trading-post service charges are exogenously given. Think of some outside agency as selecting and allocating these charges at the outset, before trading starts.[7] Interestingly, this charge is the *same* across *all* markets for a commodity, but need not be the same across different commodities—see more about the potency of this specification below.

We show that non-uniform prices in equilibrium are much more persistent than has been portrayed in previous models. Indeed, the LOP fails even in cases where conventional wisdom dictates it should not, namely, with a continuum of small agents, and infinitely many atoms. Perhaps it would be helpful at this point to spell out what the failure of the LOP is *not*. An unequal-price equilibrium in our model does not simply mean different market-clearing prices and *similar* effective after-service-charge (ASC) prices across posts for a commodity. As previously remarked, this service charge is *equal at all* posts for the same commodity. Hence, the failure of the LOP as postulated in this paper not only means different market-clearing prices, but *also different* effective ASC prices across different posts for a commodity. This is a strong result.

The intuition behind the failure of the LOP in *every* robust counterexample[8] considered in this paper is the same: what to outside observers seems like an arbitrage opportunity, is actually not for the active market participant. We explain why this is so. The large pure traders and post owners (who also trade) affect market-clearing prices nontrivially. Hence, whenever they try to take advantage of the price difference by altering their bid-and-offer decisions across any two posts, the resulting net change affects them adversely. Consider the "insignificant" agents now. By shifting his orders from one post to another, a negligible individual affects neither the equilibrium price, nor the equilibrium allocation. Yet, he still cannot profit from the price difference, due to the counterbalancing effect that is provided by the trading-post service charge. For clarity, let us contemplate one such very small agent who shifts all of his bids from the more expensive to the cheaper markets, and all of his offers from the cheaper to the more expensive posts. In doing so, he incurs charges on the *full amounts* of: (i) his bids, and; (ii) his receipts from sales, across both markets. The net gain obtained by the shift of orders is thus more than completely offset by the increase in premia payable. So, no insignificant agent has any incentive to deviate, and this unequal-price situation is indeed sustainable as an equilibrium.

In the next section, we construct and show the failure of the LOP in a model with a continuum of small agents, and finitely many large agents and trading posts. In Section 3, we extend this model to include infinitely many trading posts per commodity. In Section 4, we generalise the model in Section 3 to include infinitely many atoms. Our conclusions are summarised in Section 5. The Appendix contains all the technical proofs.

2. The failure of the LOP, Part 1: The model

In this section, we analyse a model featuring: (i) an atomless continuum of small agents; (ii) finitely many atoms, and; (iii) finitely many markets per commodity.

We consider a pure exchange economy with small agents, represented by an atomless continuum, and large agents, represented by atoms. So, we let the set of agents be denoted by $N = N_0 \cup A$, where $N_0 = (0, 1]$, and $A = \{2, \ldots, H\}$. The collection of all half-open intervals in $(0, 1]$ defined by $\mathscr{S}_0 = \{(a, b] : a, b \in N_0\}$, where $(a, b] = \emptyset$ if $b \leq a$, is a semiring. So, let ν_0 be a measure on $\mathscr{S}_0$ such that $\nu_0((a, b]) = b - a$, and denote the Carathéodory extension of ν_0 by μ_0. Let $\mathcal{N}_0$ denote the collection of all μ_0-measurable subsets of N_0 (and recall that μ_0 is in fact the Lebesgue measure when restricted to $\mathcal{N}_0$). Next, define the collection of all the subsets of A by $\mathscr{S}_A = \mathcal{P}(A)$, which is trivially a σ-algebra (and hence, a semiring). Finally, denote by μ_A the counting measure on $\mathscr{S}_A$. We may now introduce the following properties of our set of agents:

The triple $(N, \mathcal{N}, \mu)$—where $\mathcal{N}$ is the collection of all μ-measurable sets of N, and μ is an extended real-valued, σ-additive measure defined on $\mathcal{N}$—is a complete, finite measure space of agents (See Appendix, Lemmata 1 and 2). Let $\mathcal{N}_{N_0}$ denote the restriction of $\mathcal{N}$ to N_0, and $\mathcal{N}_A$ the restriction of $\mathcal{N}$ to A. Then, the measure space $\left(N_0, \mathcal{N}_{N_0}, \mu\right)$, where $\mathcal{N}_{N_0} = \mathcal{N}_0$ and $\mu = \mu_0$ when restricted to $\mathcal{N}_{N_0}$ (See Appendix, Lemma 3), is atomless, while the measure space $\left(A, \mathcal{N}_A, \mu\right)$, where $\mathcal{N}_A = \mathscr{S}_A$ and $\mu = \mu_A$ when restricted to $\mathcal{N}_A$ (See Appendix, Lemma 4), is purely atomic. Moreover, for each $i \in A$, the singleton set $\{i\}$ is an atom of the

5 In the present model, the focus is on how post owners *charge other agents* for trading at platforms that they own. We acknowledge that ideally, setting up a trading post should also be costly. However, since it is assumed that post owners are "endowed" with such posts, there is therefore no cost for them to set up a platform, nor can they *decide* how many trading posts per commodity they would like to open or shut down.

6 Thus, in addition to how their individual bids and offers directly affect their allocations, agents must also consider how their strategies affect the premia payable that accrue to them. The introduction of this service charge leads to a modification of agents' strategy sets and holdings-surfaces, such that, as opposed to Koutsougeras (1999), and, e.g., Codognato and Ghosal (2000), but similarly to Peck et al. (1992), and Koutsougeras (2003a, b), the SMG models considered in this paper are generalised games.

7 While these charges can be endogenised, in our framework we *choose* to take these as being given, such that extremely little to almost no market power is given to the post owners. Note also that these charges may instead be viewed as taxes imposed by a government. This interpretation was suggested to me by Herakles Polemarchakis.

8 More precisely, the following is true of all the counterexamples computed in this paper: *any* endowments, *and* utility functions with the same marginal rate of substitution at the consumption allocations as computed in the respective examples, would constitute equilibria with the same properties (such that the LOP still fails). This fact attests to the robustness of our counterexamples in endowment and utility spaces.