Journal of Mathematical Economics 76 (2018) 33–44

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

English auctions with ensuing risks and heterogeneous bidders

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ARTICLE INFO

Article history:
Received 26 January 2018
Accepted 4 February 2018
Available online 6 March 2018

Keywords:
English auction
Ensuing risk
Heterogeneous risk preferences
Interdependent values
Ex post equilibrium
Ex post efficiency

ABSTRACT

We establish conditions under which an English auction for an indivisible risky asset has an efficient ex post equilibrium when the bidders are heterogeneous in both their exposures to, and their attitudes toward, the ensuing risk the asset will generate for the winning bidder. Each bidder's privately known type may affect both his risk attitude and the expected value of the asset's return to the winner. An ex post equilibrium in which the winning bidder has the largest willingness to pay for the asset exists if two conditions hold: each bidder's marginal utility of income is log-supermodular, and the vector-valued function mapping the type vector into the bidders' expected values for the asset satisfies a weighted average crossing condition. However, this equilibrium need not be efficient. We show that it is efficient if each bidder's expected value for the asset is nonincreasing in the types of the other bidders, or if the bidders exhibit nonincreasing absolute risk aversion, or if the asset is riskless.

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1. Introduction

A notable feature of an English auction is the open ascending-bid process that allows bidders to infer and use the private information of the others before the auction ends. A remarkable consequence is that, in a variety of settings, an English auction has an efficient ex post equilibrium. A number of recent studies have established this in settings with heterogeneous bidders and interdependent values: Maskin (1992), Krishna (2003), Dubra et al. (2009), and Birulin and Izmalkov (2011). These are important results, given the detail-free nature and simplicity of English auctions, and the robust, belief-free nature of ex post equilibria.

In this paper we consider English auctions in environments with two additional features, both of which are prevalent in reality. First, the value the winning bidder will receive from the object may be uncertain when the auction is held. Specifically, we consider the case in which the object being sold is a risky asset that gives the winning bidder a random monetary return at some time after the auction. This return is the sum of its expected value, a function of all the bidders' types, and a random noise that is independent of the types which we refer to as an "ensuing risk". The second feature, without which the ensuing risk would be irrelevant, is that the bidders need not be risk neutral. Heterogeneity abounds: the bidders may have different utility functions for income, different expected value functions for the asset's return, and different ensuing risks. The model nests as special cases essentially all existing models of single-object auctions with unidimensional private information.

Ensuing risk is a feature of objects sold in many auctions. The value of fine wine, antiques, land, licenses, mineral rights, and takeover targets generally remains uncertain to the winner at the time an auction concludes. Eső and White (2004) provide the first analysis of auctions with ensuing risk, interdependent types, and risk averse bidders, focusing on the effect of an increase in the ensuing risk on equilibrium payoffs. Our environment generalizes theirs by allowing heterogeneity and nonmonotonic value functions, and our focus is on the existence of efficient ex post equilibria in English auctions. An important assumption in both papers, however, is that the risk is noncontractible (uninsurable). This is the case when its realization is observed only by the winning bidder, or is unverifiable for some other reason to a third-party enforcer of contracts.[1]

The addition of ensuing risk and nonlinear utility significantly alters the analysis of an English auction. A central role is played by the bidders' willingness-to-pay functions that map each type profile into the maximal amounts they would be willing to pay for the asset. Indeed, in the equilibrium of interest, the asset is always sold to the bidder with the highest willingness to pay. The existence of this equilibrium depends on the properties of the willingness-to-pay functions. In the usual setting with risk neutral bidders, a bidder's willingness-to-pay function is his value

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[1] *Contractible* ensuing risks are the focus of the literature on designing auctions with contingent payments, as recently surveyed by Skrzypacz (2013).

https://doi.org/10.1016/j.jmateco.2018.02.002
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function for the object, a primitive of the model about which any desired properties may be directly assumed. In our environment a value function is also specified for each bidder: it maps the type profile into the asset's expected return for him. However, here a bidder's willingness-to-pay function is determined jointly by his possibly nonlinear utility function, his value function, and the distribution of the ensuing risk. One contribution of this paper is to find conditions on these primitives, in particular the utility and value functions, which cause the willingness-to-pay functions to have properties that are sufficient for the existence of an ex post equilibrium in which the winner is always the bidder with the highest willingness to pay.[2]

As is generally true in the auction literature, a bidder's privately known type in this paper is unidimensional.[3] However, we allow a bidder's type to play two roles. First, it may affect the expected value of the asset to both himself and to the other bidders. This is the standard interdependency considered in the literature: a bidder's type may directly determine the expected value of the asset to himself, or it may be a signal of its value to them all. The second, more novel role for a bidder's type in this paper is that it may affect his risk attitude toward the ensuing risk. A bidder's type may thus be an argument of both his utility and value functions, as well as the value functions of the other bidders.

For each of these two roles a bidder's type may play, we make a separate assumption. First, regarding its effect on the utility function, Assumption A1 requires a bidder's marginal utility of income to be log-supermodular in his type and income. This assumption implies that a bidder's risk tolerance increases in his type. (We make no assumption about whether bidders are risk averse or loving.[4])

Second, regarding how types affect the value functions, Assumption A2 specifies that they together satisfy a "weighted average crossing condition". This condition is related to, but distinct from, the "average crossing condition" of Krishna, 2003; neither condition implies the other. Both of them amount to assuming the bidders' value functions satisfy some sort of single-crossing condition: an increase in a bidder's type increases his expected value more than it does some weighted average of the others' values. Our condition allows the weights to vary with the type vector. When the bidders are risk neutral or there is no ensuing risk, such conditions have been shown since Maskin (1992) to be sufficient for English auctions to have efficient ex post equilibria.

An increase in a bidder's type thus weakly increases both his risk tolerance (by A1) and the expected value of the asset for him (an easy implication of A2). Although the analysis is simpler and all the results hold if only one of these monotonicities is strict, it is not implausible that they both be strict. For example, suppose a bidder's type is his innate "productivity" which determines both the wealth he has accumulated to date and the expected value of the asset if he were to put it to use. It is then natural to assume that an increase in the bidder's type increases both the asset's expected value to him and his current wealth. The latter effect, assuming the bidder exhibits decreasing absolute risk aversion, implies that his risk tolerance increases in his type. So both assumed monotonicities are strict in this scenario.

Assumptions A1 and A2 together imply that the willingness-to-pay functions satisfy our weighted average crossing condition. This, we show, implies that the English auction has an ex post equilibrium in which the asset is sold to a bidder with the highest willingness to pay (Theorem 1 and Corollary 1). The proof somewhat resembles that of Theorem 2 in Krishna (2003), applied to the willingness-to-pay functions instead of the value functions. However, a notable difference is that instead of obtaining the (inverse) equilibrium strategies by applying an existence theorem to a system of ordinary differential equations, we construct them directly from the global inverse of the vector-valued willingness-to-pay function of (all) the bidders. The mathematical contribution of this paper is to show that the weighted average crossing condition implies that this global inverse function exists, using a theorem of Gale and Nikaidô (1965). (We expect this global invertibility result to be useful in other contexts as well.)

Turning to the efficiency question, we ask whether the equilibrium allocations are Pareto efficient conditional on the realized types.[5] The fact that the winning bidder has the highest willingness-to-pay for the asset does not imply in our model that the resulting allocation is ex post efficient. Because of adverse income effects and value interdependencies, the winner may be able to sell the asset to a losing bidder to their mutual benefit. We present a symmetric, two-bidder example in which this is the case whenever the bidders' types are unequal. This inefficiency is due to two factors. First, the losing bidder's willingness to pay for the asset increases in the winning bidder's type. Since the price at which he drops out is based on an underestimate of the winning bidder's type, the losing bidder's willingness to pay is larger ex post. Second, an adverse income effect is caused by the winner's increasing absolute risk aversion. His positive information rent due to winning makes him more risk averse, and hence lowers the price at which he is willing to sell the asset. The result is that ex post, the loser is willing to pay more for the asset than the lowest price the winner is willing to sell it, generating gains from trade.

However, the equilibrium is ex post efficient under additional, relatively mild conditions. Our second main result (Theorem 2) is that the equilibrium shown to exist in Theorem 1 is ex post efficient if each bidder's expected value for the asset is nonincreasing in the types of the other bidders, or if the bidders exhibit nonincreasing absolute risk aversion, or if the asset is riskless.

The rest of the paper is organized as follows. The general environment is described in Section 2. Assumptions A1 and A2 are presented in Section 3, together with their implications. Our formulation of a button English auction is presented in Section 4. Theorem 1 and Corollary 1 on existence appear in Section 5, and Theorem 2 on efficiency in Section 6. Properties of matrices that satisfy a dominant weighted average condition are derived in Appendix A, and proofs missing from the text are in Appendix B.

2. Environment

An indivisible asset is to be sold to one of several bidders using an English auction. The asset will generate a random return for the winning bidder after the auction has been held. Each bidder has a private type which can affect his risk preferences regarding the asset's ensuing risk, as well as the expected value of that risk.

The set of bidders is $N = \{1, \ldots, n\}$, where $n \geq 2$. The private type of bidder i, denoted as t_i, is an element of a compact interval $T_i = [\underline{t}_i, \bar{t}_i] \subset \mathbb{R}$. A type profile is denoted as $\mathbf{t} = (t_1, \ldots, t_n)$, an element of $\mathbf{T} := T_1 \times \cdots \times T_n$. The bidders may or may not have a common prior on the type space—we need not specify this because we are concerned with ex post equilibria.

If bidder i obtains the asset and the realized type vector is $\mathbf{t}$, the asset will generate for him a random income $v^i(\mathbf{t}) + \tilde{z}_i$. The random

2 Existence *per se* is not an issue. In any setting in which some bidder wants to win at the reserve price for any realization of the types, the English auction has an ex post equilibrium in which that bidder never drops out and all the others drop out immediately, regardless of their types.

3 Unidimensionality seems to be a necessary restriction, since efficient ex post equilibria generically do not exist if types are multidimensional (Dasgupta and Maskin, 2000; Jehiel and Moldovanu, 2001).

4 A bidder's utility for income could have convex parts because of, e.g., previously obtained financing that must be paid back, and limited liability.

5 This is the usual notion of ex post efficiency, but it does seem to presuppose that losing bidders learn the winning bidder's type. We discuss this more in Section 6.

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