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Nonlinear electrostatic behavior for two elastic parallel fixed–fixed and cantilever microbeams

Rezazadeh Ghader [a,*], Tahmasebi Ahmadali [b], Saeed Ziaei-rad [b]

[a] Mech. Eng. Dept., Urima University, Urmia, Iran
[b] Mech. Eng. Dept., Isfahan University of Technology, Isfahan 84156-83111, Iran

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ABSTRACT

In this paper, the electrostatic pull-in behavior of two elastic parallel fixed–fixed and cantilever microbeams in microelectromechanical systems (MEMS) are investigated. The nonlinear electrostatic equations are considered due to some important effects including: residual stresses, fringing field and axial stresses. Various residual stresses in two elastic parallel fixed–fixed models are considered. Step by step linearization method is used to solve the equations. The numerical results reveal that the step by step linearization method is highly efficient, and it is the easiest one to calculate the pull-in voltage. In the proposed models, the pull-in voltages are considerably decreased when compared to the pull-in voltages of simple fixed–fixed and cantilever models.

1. Introduction

Microelectromechanical devices are increasingly being integrated into electronic circuitry as their benefits become apparent. Although there are many microelectromechanical system (MEMS) designs that use piezoelectric, thermal, pneumatic, and magnetic actuation, the most popular approach in present is to use electrostatic actuation to move micromachined parts. Electrostatically actuated microstructures are also referred to as electrostatic MEMS. One of these types of devices is the microswitch.

Pull-in phenomenon is a discontinuity related to the interplay of the elastic and electrostatic forces where a potential difference is applied between two conducting structures, the structures deforms due to electrostatic forces. The elastic forces grow about linearly with displacement whereas the electrostatic forces grow inversely proportional to the square of the distance. When the voltage is increased the displacement grows until at some point the growth rate of the electrostatic force exceeds that of the elastic force and the system cannot reach a force balance without a physical contact, thus pull-in occurs. This critical voltage is known as the pull-in voltage. Because of the micro scale of the electro MEMS switches some factors, such as residual stress in thin films, fringing field effect, axial stress can influence the pull-in voltage. Some applications of determination of the pull-in voltage can be measuring the Young's modulus and the residual stress [1] and material properties [2] of MEMS switches.

However, MEMS switches suffer from a range of problems, including high driving voltage, relatively low speed and low power handling capability. In recent years, many efforts have been directed at solving these problems. Researchers have proposed a variety of methods to decrease the pull-in (actuation) voltage [3–10]. One of them involves decreasing the air gap between the fixed plate and the beam [3]. Another method involves increasing the electrostatic area [4], and a third one is decreasing the spring constant of the beams [5–10]. In this paper, to decrease the pull-in voltage, two elastic parallel fixed–fixed and cantilever models are considered. The obtained pull-in voltages are compared to the pull-in voltages of simple fixed–fixed and cantilever models. The results show that these models are very good for using in MEMS actuators structures to decrease the pull-in voltage.

The determination of the pull-in voltage and position requires the solution of a coupled electrostatic–elastic system [11]. Traditionally the pull-in analysis is done using voltage iteration which is a method of brute force. In the method the potential difference is gradually increased and for each value the coupled problem is solved iteratively. If a solution is obtained then the voltage is below the pull-in voltage otherwise the opposite is true. This scheme has no physical limitations but it is computationally very expensive. Around the pull-in position the convergence of the coupled problem may be slow. The accuracy of the scheme is determined by the step size of the scanning. Economical and reasonably accurate scanning strategies usually require some initial estimate of the pull-in voltage. Other methods for determination of the pull-in voltage have been already investigated including: (1) lumped energy model-based method [12], (2) Galerkin's method whose basis

* Corresponding author. Tel.: +98 914 145 1407; fax: +98 441 277 7022.
E-mail address: g.rezazadeh@mail.urmia.ac.ir (G. Rezazadeh).

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functions were obtained by selecting few linear undamped mode shapes, (3) discretization techniques for both microstructure and electromagnetic field using 2D and 3D models (such as BEM, FEM [13] and differential quadrature method (DQM)) [14] (4) reduced order model-based method [15], (5) meshless local Kriging method [16] and so on.

Although the number of techniques already available is quite high and they cope the need for advanced CAE tools for MEMS, the availability of a number of methods can make hard for engineers the selection of the most suitable approach, depending on the level of approximation required to determine the pull-in voltage. One of the best approaches to evaluate the pull-in voltage in MEMS switches can be the linearization of nonlinear electrostatic equation [17,18]. Because of linear nature of this equation, the solution may be very easy, fast and reliable.

The majority of this work is about the use of the step by step linearization method for two elastic parallel fixed–fixed and cantilever microbeams to investigate the improvement of the pull-in voltage due to many effects including: residual stress, fringing field and axial stress. The proposed method can be used for different residual stresses in two elastic parallel fixed–fixed models. The results show that the step by step linearization method is easier and faster than previously cited methods and can be highly efficient when used for the analysis of MEMS actuators.

2. Models descriptions and assumptions

In this paper, four different structures for microbeams are considered. Fig. 1 shows a simple fixed–fixed microbeams with thickness t, width$b$, length L and isotropic with Young's modulus E. Fig. 2 shows the two elastic parallel fixed–fixed microbeams where for the top and bottom microbeams, the geometrical and physical parameters are the same as simple one. Figs. 3 and 4 show a simple and two elastic parallel cantilever microbeams with the geometrical and physical parameters as the same as shown in Figs. 1 and 2.

Electrostatically actuated microbeams are usually modeled as continuous and prismatic straight beams, made of elastic and homogeneous material [19], with principal axes of elasticity equally directed for all sections. The above assumption allows uncoupling flexural, torsional and axial behaviors. Since the flexural behavior is here mainly considered, transverse displacement and rotation are suitable to write equilibrium equations.

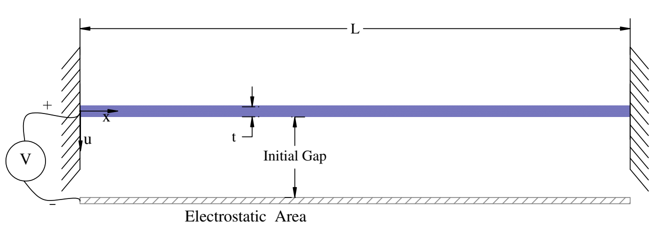

Fig. 1. Schematic of a simple fixed–fixed microbeam.

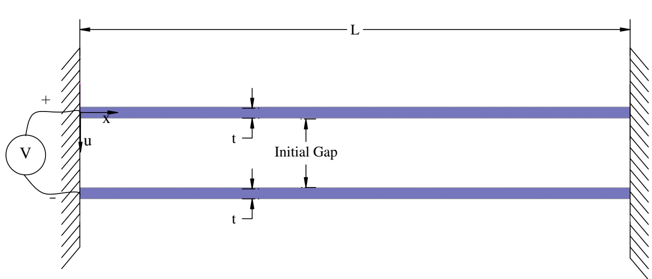

Fig. 2. Schematic of a two elastic parallel fixed–fixed microbeams.

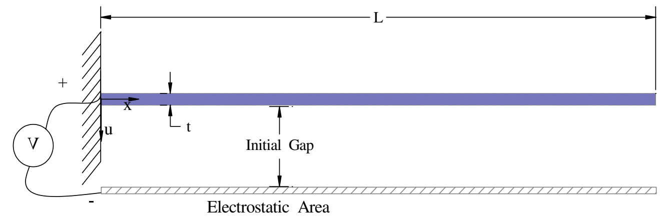

Fig. 3. Schematic of a simple cantilever microbeam.

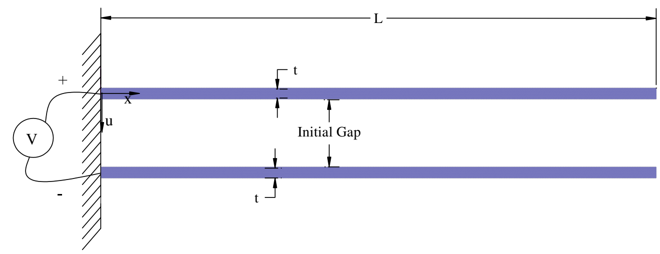

Fig. 4. Schematic of a two elastic parallel cantilever microbeams.

3. Deflection analysis

When a driving voltage is applied between the electrodes, the electrostatic force deflects the microbeam. Normally, the electrostatic force is approximately proportional to the inverse of the square of the distance between the electrodes. When the voltage exceeds the critical voltage, the microbeam suddenly pulled into the electrode. For evaluating the nonlinear deflection term and boundary conditions effects in the deflection equation, the following models are considered.

3.1. Simple fixed–fixed microbeam model

The governing differential equation for a simple fixed–fixed microbeam without consideration of the piezoelectric layers can be presented as follows [17]:

$$\frac{d^2}{dx^2}\left[\widetilde{E}I(x)\frac{d^2u}{d^2x}\right] - \left[T_r + \frac{\widetilde{E}bt}{2L}\int_0^L \left(\frac{du}{dx}\right)^2 dx\right]\frac{d^2u}{dx^2} = \frac{\varepsilon_0 bV^2}{2(g_0 - u(x))^2}(1 + F_r) \quad (1)$$

with [20]

$$T_r = \hat{\sigma}bt, \quad \hat{\sigma} = \sigma_0(1 - v)$$

The first order fringing field correction is denoted as [1]:

$$F_r = 0.65\frac{g_0 - u(x)}{b} \quad (2)$$

where $I(x)$ is the moment of inertia of the cross-sectional area; ε_0 is the permittivity of air; V is the applied voltage to the elastic parallel plates, g_0 is the initial gap between elastic parallel plates, σ_0 is the biaxial residual stress; $\hat{\sigma}$ is the effective residual stress, v is the Poisson's ratio and T_r is the residual force. For a wide beam, for which $b \geqslant 5t$, the effective modulus $\widetilde{E}$ can be approximated by the plate modulus $\frac{E}{(1-v^2)}$; otherwise $\widetilde{E}$ is the Young's modulus E.

3.2. Two elastic parallel fixed–fixed microbeams model

Considering the previous governing equation, the nonlinear governing equations for this model can be written as: