Ex-post core, fine core and rational expectations equilibrium allocations

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ABSTRACT

This paper investigates the (weak) ex-post core and its relationships to the fine core and the set of rational expectations equilibrium allocations in an oligopolistic economy with asymmetric information, in which the set of agents consists of some large agents and a continuum of small agents and the space of states of nature is a general probability space. We show that under appropriate assumptions, the (weak) ex-post core is not empty and contains the set of rational expectations equilibrium allocations. We provide an example of a pure exchange continuum economy with asymmetric information and infinitely many states of nature, in which the ex-post core does not coincide with the set of rational expectations equilibrium allocations. We also show that when our economic model contains either no large agents or at least two large agents with the same characteristics, the fine core is contained in the ex-post core.

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1. Introduction

In general equilibrium theory, the core and competitive equilibrium are two important solution concepts. For an exchange economy with complete information, the core and its relationship to the set of competitive allocations have been studied intensively in the literature (for a comprehensive survey, refer to Anderson, 1992). In the past few decades, several alternative cooperative and non-cooperative equilibrium concepts have been proposed, in the context of asymmetric information economies. The core of an economy with asymmetric information was first considered by Wilson (1978), where the concepts of coarse and fine core were proposed. In Yannelis (1991), Yannelis introduced the concept of private core, which is an analogue concept to the core for an economy with complete (and symmetric) information, and proved that under appropriate assumptions, the private core is always non-empty. Furthermore, Einy et al. (2000b, a) studied the notion of ex-post core, in the sense that an ex-post core allocation cannot be ex-post blocked by any coalition. On the other hand, Radner (1979) introduced the notion of a (Bayesian) rational expectations equilibrium by imposing the Bayesian (subjective expected utility) decision doctrine, in order to capture the information revealed by

the market clearing price. The fact that a Bayesian rational expectations equilibrium does not exist universally motivates de Castro et al. (2013) to introduce the concept of a maximin rational expectations equilibrium, by replacing the Bayesian decision-making approach of Radner with the maximin expected utility. A good survey article for the equilibrium concepts in asymmetric (or differential) information economies is Glycopantis and Yannelis (2005).

For economies with complete information, Aumann (1964) proved that competitive and core allocations coincide, provided that there is a continuum of traders. The existence of such allocations was studied by Aumann (1966) and Hildenbrand (1974). Extensions of these results to economies with asymmetric information were made by Einy et al. (2000b, 2001). In Einy et al. (2000b), Einy et al. first established some representation results on the ex-post core and the set of rational expectations equilibrium allocations. Then, these representation results together with Aumann's Core–Walras Equivalence Theorem enabled them to show that if the economy is atomless, the utility function and the initial endowment of each agent are measurable with respect to his private information, then the set of rational expectations equilibrium allocations coincides with the ex-post core. In the past few years, Bhowmik et al. have developed techniques in Bhowmik et al. (2014) to investigate the existence of rational expectations equilibria in a general model of pure exchange economies. Moreover, Bhowmik and Cao (2016) established a representation result for rational expectations equilibrium allocations in terms of the state-wise Walrasian allocations. As a rational expectations equilibrium

allocation is an interim solution concept and it takes into account the information of all other agents through market price, Bhowmik and Cao (2016) showed their result by assuming that each agent knows his initial endowment and utility. Such assumptions lead to a fact that the information revealed by prices play no role and thus, the Bayesian (maximin) rational expectations equilibrium allocations becomes almost the same as the state-wise Walrasian allocations.[1]

Our aim of this paper is to apply the results and techniques developed in Bhowmik et al. (2014) and Bhowmik and Cao (2016) to the study of the (weak) ex-post core and its relationships to the fine core and the set of rational expectations equilibrium allocations. We consider an oligopolistic economy with asymmetric information, in which the set of agents consists of some large agents and a continuum of small agents. The uncertainty is modelled by a general probability space of states of nature, in which each agent is characterized by a state-dependent utility function, a state-dependent initial endowment, an information partition and a prior belief. Firstly, we extend the notion of ex-post core and establish results on the existence and characterization of the (weak) ex-post core, which can be regarded as extensions of the corresponding result in Einy et al. (2000b) to a framework with infinitely many states of the nature. The proof of these results rely on the measurability of Walrasian equilibrium correspondence with respect to the information structure in the economy (see Theorems 3.3 and 3.6). With helps of the main result in Bhowmik and Cao (2016) and Aumann's Core–Walras Equivalence Theorem, we can conclude that Bayesian (maximin) rational expectation equilibrium allocations are contained in the ex-post core. This is a version of the first fundamental theorem of social welfare for large economies with asymmetric information. However, contrary to the equivalence result in Einy et al. (2000b), we provide an example of a continuum economy with asymmetric information and infinitely many states of nature, in which the ex-post core strictly contains all rational expectations equilibrium allocations. This means that the Core–Walras equivalence may fail in a continuum economy with asymmetric information when it has infinitely many states of nature. Secondly, we show that under appropriate assumptions and the additional assumptions that there are only finitely many different information structures and all information is the joint information of agents, the fine core is contained in the ex-post core. This extends the corresponding result in Einy et al. (2000a). To obtain this result, following Greenberg and Shitovitz (1986), we associated an atomless economy with our oligopolistic economy so that all large agents are broken into a continuum of small agents with the same characteristics.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework and outlines the basic model. We also provide some results on several correspondences associated with our basic model. These correspondences form the major part of our tool kits. Section 3 introduces the notions of ex-post core and weak ex-post core. We prove some results on representation of the (weak) ex-post core, which can be used to establish the relationships between the (weak) ex-post core and the set of rational expectations equilibrium allocations. Section 4 studies the relationship between the ex-post core and the fine core. Section 5 summarizes the major achievement of this paper and also includes some further remarks and relevant questions. Finally, the proofs of two technical lemmas are provided in Appendices A and B.

2. The model and associated correspondences

In this section, we describe a basic model of a pure exchange mixed economy with asymmetric information.

2.1. The model

We consider a pure exchange economy $\mathscr{E}$ with asymmetric information. The exogenous uncertainty is described by a probability space $(\Omega, \mathscr{F}, \mathbb{P})$, where Ω is a set denoting all possible states of nature, the σ-algebra $\mathscr{F}$ denotes possible events, and $\mathbb{P}$ is a complete probability measure. The space of agents is a measure space (T, Σ, μ) with a complete, finite and positive measure μ, where T is the set of agents, Σ is the σ-algebra of measurable subsets of T whose economic weights on the market are given by μ. Since $\mu(T) < \infty$, a classical result in measure theory claims that T can be decomposed into the union of two parts: one is atomless and the other contains at most countably many atoms, that is, $T = T_0 \cup T_1$, where T_0 is the atomless part and $T_1 = \bigcup_{n \geq 1} A_n$ is the union of at most countably many μ-atoms A_n's, refer to Nielsen (1997, p. 155). Agents in T_0 are called "*small agents*", who are non-influential agents (the price takers). Agents in T_1 are called "*large agents*", who are influential ones (the oligopolies). According to a standard interpretation, we can think that each A_n arises from a group of small identical agents that decide to join and to act on the market only together. As consequence of such agreements, no proper subcoalitions of the group are possible and then the group is identified with an atom of μ. The commodity space is the ℓ-dimensional Euclidean space $\mathbb{R}^\ell$. For $\lambda > 0$, $B(0, \lambda)$ denotes the ball in $\mathbb{R}^\ell$ centred at 0 with radius λ. The partial order on $\mathbb{R}^\ell$ is denoted by $\leq$. More precisely, for any two vectors $x = (x_1, \dots, x_\ell)$ and $y = (y_1, \dots, y_\ell)$ in $\mathbb{R}^\ell$, we write $x \leq y$ (or $y \geq x$) if $x_k \leq y_k$ for all $1 \leq k \leq \ell$. Furthermore, we write $x < y$ (or $y > x$) when $x \leq y$ and $x \neq y$, and $x \ll y$ (or $y \gg x$) when $x_k < y_k$ for all $1 \leq k \leq \ell$. Let $\mathbb{R}^\ell_+ = \{x \in \mathbb{R}^\ell : x \geq 0\}$, and let $\mathbb{R}^\ell_{++} = \{x \in \mathbb{R}^\ell_+ : x \gg 0\}$. In each state, the *consumption set* for every agent $t \in T$ is $\mathbb{R}^\ell_+$. Each agent $t \in T$ is characterized by a quadruple $(\mathscr{F}_t, U(t, \cdot, \cdot), a(t, \cdot), \mathbb{P}_t)$, where

– $\mathscr{F}_t$ is the σ-algebra generated by a measurable partition Π_t of Ω (i.e., $\Pi_t \subseteq \mathscr{F}$) representing the *private information* of agent t,

– $U(t, \cdot, \cdot) : \Omega \times \mathbb{R}^\ell_+ \to \mathbb{R}$ is the *state-dependent utility function* of agent t,

– $a(t, \cdot) : \Omega \to \mathbb{R}^\ell_+$ is the *state-dependent initial endowment* of agent t, and

– $\mathbb{P}_t$ is a probability measure on $\mathscr{F}$, giving the *prior belief* of agent t.

The quadruple $(\mathscr{F}_t, U(t, \cdot, \cdot), a(t, \cdot), \mathbb{P}_t)$ is often known as the *characteristics* of agent t. Two agents are said to be the *same type* if they have the same characteristics. Formally, the economy $\mathscr{E}$ can be expressed by

$$\mathscr{E} = \{(\Omega, \mathscr{F}, \mathbb{P}); \ (T, \Sigma, \mu); \ \mathbb{R}^\ell_+; \ (\mathscr{F}_t, U(t, \cdot, \cdot), a(t, \cdot), \mathbb{P}_t)_{t \in T}\}.$$

In the complete information Arrow–Debreu–McKenzie model, prices are vectors in $\mathbb{R}^\ell_+ \setminus \{0\}$. Following the standard treatment in the literature (e.g., see Aumann, 1966), price vectors are normalized so that their sum is 1.

In this paper, we use the symbol Δ to denote the simplex of normalized price vectors, i.e.,

$$\Delta = \left\{ p \in \mathbb{R}^\ell_+ : \sum_{h=1}^\ell p^h = 1 \right\}.$$

Put $\Delta_+ = \Delta \cap \mathbb{R}^\ell_{++}$. Throughout the paper, Δ and Δ_+ are equipped with the relative Euclidean topology. A *price system* of $\mathscr{E}$ is an $\mathscr{F}$-measurable function $\pi : \Omega \to \Delta$, where Δ is equipped with the Borel structure $\mathscr{B}(\Delta)$ generated by the relative Euclidean topology.

Let $\sigma(\pi)$ be the smallest σ-algebra contained in $\mathscr{F}$ and generated by a price system π. Intuitively, $\sigma(\pi)$ represents the information revealed by π. The combination of agent t's private information $\mathscr{F}_t$ and the information revealed by the price system

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