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Perturbed utility and general equilibrium analysis

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ABSTRACT

We study general equilibrium theory of complete markets in an otherwise standard economy with each household having an additive perturbed utility function. Since this function represents a type of stochastic choice theory, the equilibrium of the corresponding economy is defined to be a price vector that makes its mean expected demand equal its mean endowment. We begin with a study of the economic meaning of this notion, by showing that at any given price vector, there always exists an economy with deterministic utilities whose mean demand is just the mean expected demand of our economy with additive perturbed utilities. We then show the existence of equilibrium, its Pareto inefficiency, and the upper hemi-continuity of the equilibrium set correspondence. Specializing to the case of regular economies, we finally demonstrate that almost every economy is regular and the equilibrium set correspondence in this regular case is continuous and locally constant.

1. Introduction

The classical theory of general equilibrium, as initiated in Walras (2003) and culminated in Debreu (1959), postulates that an individual preference be deterministic. The standing of this postulate has been submitted to test by a number of empirical studies, as for instance Davidson and Marschak (1959). The result has cast grave doubt on its standing. This leads Block and Marschak (1960, p. 97) to remark that "there is a need to substitute 'stochastic consistency of choices' for 'absolute consistency of choices.'" We are then confronted with two questions: First, how to construct a utility theory that can best represent the stochastic consistency and, second, under this new theory, in what sense and to what extent we can reconstitute the classical general equilibrium theory.

For the first question a multitude of attacks have been made, beginning with Thurstone (1927) through Marschak's work mentioned above to more recent work of Fudenberg et al. (2015) and Gul et al. (2014). In broad outline they fall under two main headings: random utility maximization model and additive perturbed utility (APU) model, with all but Fudenberg et al. (2015) belonging to the first class. These two types of models have overlap but neither nests the other.

As regards the second question the theory of general equilibrium under random utility maximization model has been studied with the pioneering work of Hildenbrand (1971). The main objective here is to examine the statistical properties of the total excess demand and of the equilibrium price vectors (cf. Bhattacharya and Majumdar, 1973). A typical result along this line of research is exemplified by a kind of a central limit theorem: The total excess demand per capita of a random economy tends, under suitable conditions, to be normally distributed when the number of households in it increases without limit.

With this background the present paper undertakes to investigate general equilibrium theory under the APU model of Fudenberg et al. (2015). Our objective is to reconstitute as many aspects as possible of the classical general equilibrium theory. More specifically we shall examine what is an appropriate notion of equilibrium, its existence, efficiency, determinacy, and the properties of the equilibrium set correspondence. The major contrast with the research referred to in the last paragraph is that the total excess demand no longer forms a random variable and there is accordingly no way to study its statistical properties; but instead APU provides us with a possibility to study, under stochastic choice theory, non-convex economies and the efficiency of their equilibria. In fact, the implicit theme underlying the current investigation is that randomization supplies to us a means to handle in a satisfactory manner non-convex economies: it helps to restore some results that fail to hold for non-convex economies with deterministic utilities (cf. Section 4).

We begin in Section 2 with a review of the APU model. Since it applies only to simple lotteries (i.e. lotteries with finite support), the proper framework for our purpose is Mas-Colell (1977a)'s model of indivisible commodities. Specifically we assume all commodities are indivisible but one, which guarantees that there is, at any strictly positive price vector, only finitely many feasible points

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on the budget line, and focus our attention on economies with a continuum of households. Let $\mathbb{X}$ be the corresponding consumption space and $\mathcal{M}(\mathbb{X})$ the set of simple lotteries on it. For any $\phi \in \mathcal{M}(\mathbb{X})$ which has $\{x^1, \dots, x^n\}$ as its support and ϕ_i as the probability of x^i, APU measures the utility of ϕ in accordance with

$$U(\phi) = \sum_{i=1}^{n} [u(x^i)\phi_i - c(\phi_i)],$$

where u is a function on $\mathbb{X}$ and c a function on $[0, 1]$. Observing that APU is not continuous with respect to the weak topology on $\mathcal{M}(\mathbb{X})$, we set out to establish a different topology such that the desired continuity is obtained, and show that $\mathcal{M}(\mathbb{X})$ endowed with the new topology is separable and locally compact. We continue with a study of the continuity of the budget set correspondence and the 'demand function,'and close Section 2 with a definition of an allocation as well as its feasibility and Pareto efficiency.

In Section 3 we define the notion of equilibrium for an economy E, namely, as a price vector that makes the mean (or total) expected demand of E equal its mean endowment. We begin with a study of the economic meaning of this notion, by showing that at any given price vector, there always exists an economy with deterministic utilities whose mean demand is just the mean expected demand of E. The difference with the model of Hildenbrand (1971) lies in the fact that the resultant deterministic economy varies with the given price vector. We then show that an equilibrium exists under favorable conditions but is nevertheless not Pareto efficient in general. We end up the section with a result on the upper hemi-continuity of the equilibrium set correspondence.

Aiming to study the determinacy of equilibria we proceed in Section 4 to examine a special type of economies, i.e. regular economies, of which the definition is the same as in the case of deterministic utilities. By analogy with the latter case we study a set of economies which share the same utility structure but differ in endowment structure. We first set up a measure on the set of all admissible initial endowments and then show that almost every economy is regular. We conclude the section and also the paper by establishing, in the circumstance of regular economies, the continuity of the equilibrium set correspondence and its local constancy. This result is interesting by noting that no geometric restriction is placed on u, i.e. it is not required to be concave. Without this requirement the result, as has been proved by Mas-Colell (1977b), would fail to hold for economies with deterministic utilities.

2. The model

We assume given l (≥ 2) commodities on the market with only one of them perfectly divisible and all the others indivisible. Since the divisible commodity is usually taken to be money, it is economically reasonable to assume that the demand of the indivisible commodities does not vanish. Let

$$\mathbb{Z}_+^{l-1} = \{(z_1, \dots, z_{l-1}) : z_i \text{ is an integer for every } i \text{ and } z_j > 0 \text{ for some } j\}; \tag{1}$$

then we can take as the consumption space $\mathbb{X} = \mathbb{Z}_+^{l-1} \times \mathbb{R}_+$, wherein $\mathbb{R}_+$ stands for the set of nonnegative real numbers. On the other hand we take $\mathbb{R}_{++}^l$ as the space of initial endowments. This contrast with the consumption space, of which use will be made in Section 4, makes economic sense and can be understood from the viewpoint of aggregation across goods (cf. Varian, 1992, Section 9.3). Take the car market for instance. It is sometimes convenient and reasonable to model the household's choice of a car without distinguishing whether the car is new or old or what brand it is. Suppose the "average price" of the car is p. Then the value of the endowment of a household with a new car in his hand will be above p, and that of a household with a used car below p. This provides the justification for taking $\mathbb{R}_{++}^l$ as the space of initial endowments.

Throughout the paper we shall use $\| \cdot \|$ to denote the 1-norm of a vector. Let $\mathbb{P}$ be the price simplex, i.e.

$$\mathbb{P} = \{p \in \mathbb{R}_+^l : \|p\| = 1\},$$

and $\mathbb{P}_{++}$ its relative interior, $\partial\mathbb{P}$ its relative boundary. Recall that by a preference being incomplete we understand the existence of at least one pair of commodity bundles that are not comparable to each other. In this paper we assume that the households involved all have incomplete but strongly monotone preferences on $\mathbb{X}$ (see von Neumann and Morgenstern (1947, pp. 28–29) and Shapley and Baucells (1998) for the reasons a household may possibly have incomplete preference). This means that at a given price level $p \in \mathbb{P}$, the household with initial endowments e will choose from

$$B(p) = \{x \in \mathbb{X} : px = pe\}, \tag{2}$$

but he may nevertheless not be able to compare x^1 and x^2 for some $x^1, x^2 \in B(p)$. If, and this will be assumed in the sequel, the household is forced to decide between x^1 and x^2, we postulate that he will do so in a random fashion.

To describe this random behavior let $\mathcal{M}(\mathbb{X})$ be the set of simple lotteries on $\mathbb{X}$, i.e. probability measures with a finite support. Noting that $B(p)$ is a finite set on $\mathbb{P}_{++}$, the household's choice among $B(p)$ can therefore be formalized by an element of $\mathcal{M}(\mathbb{X})$. Since a great deal of notation will be introduced presently, it seems appropriate here to make a convention on the symbolism: we shall use ϕ, ϕ^1, ϕ^2 and the like to denote generic elements of $\mathcal{M}(\mathbb{X})$; for ϕ and for any other vector that will appear in this paper, we shall use superscripts to distinguish between different vectors and subscripts to indicate the components of a vector.

According to Fudenberg et al. (2015) the random behavior of the household can, as stated in the introduction, be represented by a perturbed utility function U from $\mathcal{M}(\mathbb{X})$ to $\mathbb{R}$:

$$U(\phi) = \sum_{i=1}^{n} [u(x^i)\phi_i - c(\phi_i)], \tag{3}$$

where $\phi \in \mathcal{M}(\mathbb{X})$ has $\{x^1, \dots, x^n\}$ as its support and ϕ_i as the probability of x^i; $u : \mathbb{X} \to \mathbb{R}$ is continuous and strictly increasing, and c is strictly convex on $[0, 1]$ and continuously differentiable on $(0, 1)$. For any $\phi^1, \phi^2 \in \mathcal{M}(\mathbb{X})$, ϕ^1 is preferred to ϕ^2 if and only if $U(\phi^1) \geq U(\phi^2)$. Since U is defined only for simple lotteries and $B(p)$ is finite only when $p \in \mathbb{P}_{++}$, to make the equilibrium price vector (to be defined later on) a member of $\mathbb{P}_{++}$, we assume

$$\lim_{\|x\| \to \infty} u(x) = +\infty.$$

Let $\mathcal{U}$ be the set of all U with the corresponding (u, c) fulfilling the requirements above.

We now define the notion of an economy under perturbed utility. To this end we need a topology on $\mathcal{U}$. By analogy with case of deterministic utility, we identify each U with the set

$$\{(\phi^1, \phi^2) : U(\phi^1) \geq U(\phi^2)\},$$

and impose on $\mathcal{U}$ the topology of closed convergence. To make this precise however we demand a topology on $\mathcal{M}(\mathbb{X})$: The usual way is to take the weak topology, but it is not a natural one in the current context because U is not continuous with respect to it. To see this take $l = 2$ and ϕ to be the Dirac measure at $(1, 1)$. Let ϕ^n be the measure with two-element support $\{(1 - 1/n, 1 - 1/n), (1 + 1/n, 1 + 1/n)\}$, each with equal probability. Then as is easily verified, ϕ^n converges weakly to ϕ, but, due to the strict convexity of c, $U(\phi^n) \nrightarrow U(\phi)$.

In addition to the continuity of U we require for later considerations the topology on $\mathcal{M}(\mathbb{X})$ to be such that makes $\mathcal{M}(\mathbb{X})$