Journal of Mathematical Economics 66 (2016) 40–51

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

Analysis of information feedback and selfconfirming equilibrium

P. Battigalli, S. Cerreia-Vioglio, F. Maccheroni *, M. Marinacci

Università Bocconi, Italy
IGIER, Italy

ARTICLE INFO

Article history:
Received 6 October 2015
Received in revised form
21 May 2016
Accepted 15 July 2016
Available online 27 July 2016

Keywords:
Selfconfirming equilibrium
Conjectural equilibrium
Information feedback
Ambiguity aversion
Partially specified probabilities

ABSTRACT

Recent research emphasizes the importance of information feedback in situations of recurrent decisions and strategic interaction, showing how it affects the uncertainty that underlies selfconfirming equilibrium (e.g., Battigalli et al., 2015, Fudenberg and Kamada, 2015). Here, we discuss in detail several properties of this key feature of recurrent interaction and derive relationships. This allows us to elucidate different notions of selfconfirming equilibrium, showing how they are related to each other given the properties of information feedback. In particular, we focus on Maxmin selfconfirming equilibrium, which assumes extreme ambiguity aversion, and we compare it with the partially-specified-probabilities (PSP) equilibrium of Lehrer (2012). Assuming that players can implement any randomization, symmetric Maxmin selfconfirming equilibrium exists under either "observable payoffs," or "separable feedback." The latter assumption makes this equilibrium concept essentially equivalent to PSP-equilibrium. If observability of payoffs holds as well, then these equilibrium concepts collapse to mixed Nash equilibrium.

1. Introduction

In a selfconfirming equilibrium (SCE), agents best respond to confirmed, but possibly incorrect, beliefs. The notion of SCE captures the rest points of dynamics of strategies and beliefs in games played recurrently (see, e.g., Fudenberg and Levine, 1993b; Fudenberg and Kreps, 1995; and Gilli, 1999). Battigalli et al. (2015) (henceforth BCMM) define a notion of selfconfirming equilibrium whereby agents have non-neutral attitudes toward model uncertainty, or ambiguity.[1] The SCE concept of BCMM encompasses the traditional notions of conjectural equilibrium (Battigalli, 1987; Battigalli and Guaitoli, 1988) and selfconfirming equilibrium (Fudenberg and Levine, 1993a) as special cases, taking as given the specification of an ex post information structure, or information feedback. Specifically, the information feedback function describes what an agent can observe *ex post*, at the end of the stage game which is being played recurrently. The properties of information feedback determine the type of partial-identification problem faced by a player who has to infer the co-players' strategies from observed data. This, in turn, shapes the set of selfconfirming equilibria.

We define several properties of information feedback, we study their relationships, and we illustrate them through the analysis of equilibrium concepts. Specifically, we focus on Maxmin SCE, which assumes an extreme form of ambiguity aversion, and its relationships with other equilibrium concepts. We also deviate from BCMM by allowing players to delegate their choices to arbitrary randomization devices. Three properties of information feedback play a prominent role in our analysis: (i) "observable payoffs" means that each player observes his own realized utility, (ii) "own-strategy independence of feedback" means that inferences about the strategy profile played by the opponents do not depend on one's own adopted strategy; (iii) "separable feedback" is a strengthening of own-strategy independence: inferences about the strategy of each opponent are independent of how other agents play. While (i) is a natural property that holds in many applications, we argue that (ii)–(iii) are very strong properties of feedback. BCMM show that, if payoffs are observable, then the traditional ambiguity-neutral SCE concept is a refinement of Maxmin SCE; hence, ambiguity aversion (weakly) expands the equilibrium set. On the other hand, under observable payoffs and own-strategy independence of feedback every SCE concept is equivalent to mixed Nash equilibrium.

We show that all games with separable feedback have a "symmetric" Maxmin SCE in mixed strategies.[2] We observe that

* Correspondence to: Università Bocconi, Via Roentgen, 1, 20136, Milan (MI), Italy.
E-mail address: fabio.maccheroni@unibocconi.it (F. Maccheroni).

[1] For a discussion on the literature of choice under ambiguity, see the surveys of Gilboa and Marinacci (2013), and Marinacci (2015).

[2] "Symmetric" refers to the population-game scenario that we use to interpret the SCE concept: all the agents in the same player role use the same strategy.

http://dx.doi.org/10.1016/j.jmateco.2016.07.002
0304-4068/

games with separable feedback have a canonical representation as games with partially specified probabilities (PSP) in the sense of Lehrer (2012). Under this representation, our symmetric Maxmin SCE is equivalent to the equilibrium concept put forward by Lehrer. We also show that – under the (strong) assumption that all randomizations are feasible – Maxmin SCE is a refinement of ambiguity-neutral SCE. Our results imply that, in the canonical representation of a game with separable feedback, Lehrer's PSP equilibrium is a refinement of ambiguity-neutral SCE, and that under observability of payoffs it is equivalent to mixed Nash equilibrium.

The rest of the paper is organized as follows. Section 2 describes games with feedback and the partial identification correspondence; Section 3 analyzes the properties of information feedback and their consequences for the identification of co-players' behavior; Section 4 defines SCE concepts and relates them to each other and to Nash equilibrium; Section 4.3 analyzes existence of Maxmin SCE; Section 5 relates information feedback to partially specified probabilities; Section 6 further discusses our assumptions and results, and relates to the literature.

2. Games with feedback and partial identification

Throughout the analysis, we consider agents who play the strategic form of a finite game Γ in extensive form with perfect recall and no chance moves. The extensive-form structure shapes feedback and is relevant for the analysis of its properties. In this section, we introduce the key elements of games with feedback (2.1), we present the population-game backdrop of our analysis (2.2), and we define the identification correspondence (2.3).

2.1. Games with feedback

To define games with feedback, we start from a *finite* game in extensive form Γ. We defer the details of the extensive-form representation to Section 3, where we analyze the properties of feedback. Here we use only the following key primitive and derived elements of the game:

- I is the set of players *roles* in the game;
- Z is the finite set of *terminal nodes*;
- $u_i : Z \to \mathbb{R}$ is the *payoff* (vNM utility) *function* of player i;
- $S = \times_{i\in I} S_i$ is the finite set of *pure-strategy* profiles;
- $\zeta : S \to Z$ is the *outcome function*;
- $(I, (S_i, U_i)_{i\in I})$ is the *strategic form* of Γ, that is, for each $i \in I$ and $s \in S$, $U_i(s) = u_i(\zeta(s))$; as usual, U_i is multi-linearly extended to $\times_{j\in I} \Delta(S_j)$.

Following Battigalli (1987), we specify, for each player role $i \in I$, a *feedback function*

$$f_i : Z \to M,$$

where M is a *finite* set of "messages" representing what i can observe ex post about the path of play.[3] For instance, suppose that u_i is a monetary payoff function (or a strictly increasing function of the monetary payoff of i) and that i only observes ex post how much money he got, then $M \subseteq \mathbb{R}$ is a set of monetary outcomes and $f_i = u_i$. This example shows that, in our setup, the feedback function f_i does not necessarily reflect what a player remembers about the game just played; but we will introduce a property, called "ex post perfect recall", that requires just this. Another example is the feedback function assumed by Fudenberg and Levine (1993a): Each player i observes ex post the whole path of play. In this case, f_i is any injective function (e.g., the identity on Z).

A *game with feedback* is a tuple

$$(\Gamma, f) = (\Gamma, (f_i)_{i\in I}).$$

The *strategic-form feedback function* of i is $F_i = f_i \circ \zeta : S \to M$. This, in turn, yields the pushforward map $\hat{F}_i : \times_{j\in I} \Delta(S_j) \to \Delta(M)$ defined by

$$\hat{F}_i(\sigma)(m) = \sum_{s\in F_i^{-1}(m)} \prod_{j\in I} \sigma_j(s_j),$$

which gives the probability that i observes message m as determined by the mixed-strategy profile σ. We (informally) assume that each player i knows (1) the game tree and information structure (which determine S and ζ), (2) his feedback function f_i (hence his strategic-form feedback function F_i), and (3) his payoff function u_i. On the other hand, common knowledge of (Γ, f) is not relevant for our analysis, because SCE is not meant to capture inferences based on strategic reasoning.

2.2. Random matching and feasible strategies

We assume (informally) that the strategic form of game with feedback (Γ, f) is played recurrently by a large population of agents, partitioned according to the player roles $i \in I$ (male or female, buyer or seller, etc.). Agents drawn from different sub-populations are matched at random, play, get feedback according to f, and then are separated and re-matched to play again. The large-populations backdrop of our analysis is important to justify the assumption that, in steady state, non-myopic agents maximize their instantaneous expected payoff.[4] Furthermore, we assume that agents can covertly and credibly commit to play any mixed strategy.[5]

The exact details of the matching process are not important as long as the following condition is satisfied: If everyone keeps playing the same strategy, the co-players' strategy profile faced by each agent at each stage is an i.i.d. draw with probabilities given by the statistical distribution of strategies in the co-players' sub-populations. This is consistent with Nash's mass action interpretation of equilibrium (Weibull, 1996).

Our assumptions about (covert) commitment are instead important and restrictive. According to the main equilibrium concept formally defined below, Maxmin SCE, agents are ambiguity averse. Two issues arise. First, it is known that ambiguity-averse agents are dynamically inconsistent; therefore, they may be unwilling to implement contingent choices implied by some pure strategy they deem ex ante optimal (see the discussion in BCMM and the references therein). Second, even in a simultaneous-move game, an ambiguity-averse agent may not want to implement the realization of an ex ante optimal mixed strategy. Therefore, we are assuming that agents truly and irreversibly delegate their choices in Γ to some device implementing a pure or mixed strategy. We maintain such strong assumptions only for expositional simplicity. We want to focus on the properties of feedback and their consequences. The fact that players obtain feedback about the terminal node of an extensive-form game gives structure and makes the analysis more interesting. Taking dynamic incentive

3 See also Battigalli and Guaitoli (1988). Assuming a common finite set of messages is without loss of generality: let $M = \bigcup_{i\in I} f_i(Z)$.

4 Alternatively, with a fixed set of players, we should either assume perfect impatience, or look at equilibria of repeated games with imperfect monitoring, in the spirit of Kalai and Lehrer (1995). See the discussion in the survey by Battigalli et al. (1992).

5 Commitment is covert because it is not observed by other agents. Covert commitment is relevant when agents are dynamically inconsistent.