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Decentralized Pricing and the equivalence between Nash and Walrasian equilibrium

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ABSTRACT

We introduce, in the standard exchange economy model, market games in which agents use private prices as strategies. We give conditions on the game form that ensure that the only strict Nash equilibria of the game are the competitive equilibria of the underlying economy. This equivalence result has two main corollaries. First, it adds to the evidence that competitive equilibria can be strategically stable even in small economies. Second, it implies that competitive equilibria have good local stability properties under a large class of evolutionary learning dynamics.

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1. Introduction

In his *Elements of Pure Economics* (see Walras, 1874), Walras introduces two descriptions of the price adjustment process in a market economy. On the one hand, he puts forward the *tâtonnement* as a "theoretical" model. On the other hand, he emphasizes that in practice, the driver of price adjustment is *free competition*. He characterizes the latter by three conditions (see Dockès and Potier, 2005): free market entry and exit, freedom to set prices and freedom to set production levels.

This paper investigates a game theoretic model of price formation that exhibits the characteristics of free competition in the latter sense. Our key behavioral assumption is that individual agents set prices in a decentralized manner. Our main result is to show the equivalence between Nash and general equilibria in this context. More precisely, we consider a standard exchange economy in which each agent has a trade strategy that consists of a vector of *private prices* for the goods he is endowed with and the goods he consumes. These private prices represent the prices at which the agent is willing to sell the goods he supplies to the market and the maximum prices he is willing to pay for the goods he demands from the market. We consider that agents strategically update their private prices in order to improve their competitive position on different markets. The historically inclined reader will note that this approach closely matches Walras' original description of free competition:

> *As buyers, traders make their demand by outbidding each other. As sellers, traders make their offers by underbidding each other.... The markets that are best organized from the competitive standpoint are those in which...the terms of every exchange are openly announced and an opportunity is given to sellers to lower their prices and to buyers to raise their bids* (Walras, 1874, paragraph 41).

We say that a profile of private prices is *uniform* if all the agents in the economy use the same private prices. We then identify general equilibria of the economy with uniform price profiles for which the common price is a market equilibrium price. Our main result is that these general equilibrium price profiles are the only strict Nash equilibria of the model. This result holds provided that competition is effective in the sense that: (i) when there is positive excess supply in one period, a seller who failed to secure a transaction can gain by slightly undercutting his competitors' prices, (ii) when there is positive excess demand, a buyer who failed to secure a transaction can increase his utility by slightly outbidding his competitors.

The equivalence between general market equilibria and strict Nash equilibria in our model has two main corollaries. First, it adds to the evidence reviewed below that competitive equilibria can be strategically stable even in small economies. Second, it is a well-known result in evolutionary game theory (see Weibull, 1995;

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Gintis, 2009) that in multi-population games, strict Nash equilibria are the only asymptotically stable points of the replicator dynamic, and more generally of any monotone dynamic. Hence, if we treat an exchange economy as the stage game of an evolutionary process in which each agent's initial inventory is replenished in each period, general equilibria will be the only asymptotically stable states.

Our contribution is related to the large body of literature that focuses on the strategic and evolutionary foundations of general equilibrium. A first strand of literature (see in particular Rubinstein and Wolinsky, 1985, Gale, 1986a,b, McLennan and Sonnenschein, 1991, and Kunimoto and Serrano, 2002) builds on models of bargaining à la (Rubinstein, 1982) to provide strategic foundations to Walrasian equilibrium. It considers agents who are matched in a sequence of pairwise interactions during which they bargain their endowments and decide whether to leave or to stay in the market. The main result in this literature is that agents exit the market when they reach their Walrasian allocation. Hence this literature provides mechanisms that entail the stability of Walrasian allocations. However these mechanisms do not plausibly represent actual market institution and do not address the stability of equilibrium resulting from price dynamics because prices are absent in these models.

In the literature on strategic market games pioneered by Shapley and Shubik (1977) and surveyed in Giraud (2003), institutions are central. Prices are determined at trading posts at which strategically determined nominal demands and real supplies are confronted. Early contributions in this strand of literature have focused on large economies. Their main result is that, as the economy is increasingly replicated, the set of Nash equilibria of the strategic market game converges to the set of general market equilibria of the underlying economy. Our contribution is more closely related to the subsequent literature that has focused on strategic stability in small economies. Peck and Shell (1990) consider a market game model à la Shapley–Shubik in which traders may make arbitrary short sales. This possibility of short sales fosters liquidity and market competition, allowing competitive equilibria to emerge even in small economies. An alternative approach to increase liquidity, pursued by Ghosal and Morelli (2004), is to allow for retrading. Competitive equilibria can then be supported as allocations of the market game in the infinite-time limit. Hence, market games à la Shapley–Shubik yield very similar conclusions to those of the present paper about the strategic stability of competitive equilibria. The key difference is that we consider a decentralized price-formation process in which prices are set by individual agents, whereas in the market game literature prices are set centrally at trading posts. The importance of such differences in the price adjustment mechanism is strongly emphasized in Kumar and Shubik (2004). Additionally, whereas our result is an equivalence, competitive equilibria generally form a strict subset of the set of Nash equilibria of market games. Yet, recent experimental results obtained by Duffy et al. (2011) show that subjects have a tendency to coordinate on efficient Nash equilibria rather than on Pareto inferior ones. Moreover, as the number of subjects participating in the market game increases, the Nash equilibrium that experimental subjects achieve approximates the associated competitive equilibrium of the underlying economy.

These experimental results on market games are complementary to the ones obtained in continuous double-auction experiments (see Smith, 1982; Asparouhova et al., 2011, for surveys of the relevant literature), which give strong support for competitive outcomes.[1] Our approach is closely related to the theoretical literature developed to explore the stability of competitive equilibria in continuous double-auctions (Friedman, 1991; Easley and Ledyard, 1993; Gjerstad and Dickhaut, 1998; Lesourne et al., 2006). First and foremost, private prices are determined by individual traders in both cases. Second, some of the examples of exchange mechanisms considered below are very similar to double auctions. Third, consistently with the evolutionary dimension of our work, the continuous double-auction literature generally consider traders as myopic.

From this last perspective, our approach relates to the evolutionary game theory literature on market dynamics: Vega-Redondo (1997) analyzes the convergence to the Walrasian equilibrium in a Cournot oligopoly where firms update quantities in an evolutionary fashion, Alós-Ferrer et al. (2000) provide an evolutionary model of Bertrand competition, Serrano and Volij (2008) study the stability of Walrasian equilibrium in markets for indivisible goods, and several contributions (Mandel and Botta, 2009; Kim and Wong, 2011; Mandel and Gintis, 2014), investigate evolutionary dynamics in specific exchange economies. Though less precise on dynamical aspects, our contribution provides results that are more generic than those of the existing literature as it applies to a broad class of exchange economies.

The rest of the paper is organized as follows. Section 2 introduces our model economy. Section 3 defines a class of market games based on private prices in this economy and gives necessary conditions for the stability of equilibrium. Section 4 analyzes in greater detail the necessary conditions for competition to entail a stable price adjustment process. In Section 5, we illustrate numerically the extension of our results to Markovian price adjustment processes. Section 6 concludes.

2. The Walrasian economy

We consider an economy with a finite set of goods $G = \{1, \ldots, n\}$, and a finite set of agents $A = \{1, \ldots, m\}$. Each agent $i \in A$ has consumption set $\mathcal{X} = \mathbf{R}^n_+$, a utility function $u_i\colon \mathcal{X} \to \mathbf{R}_+$ and an initial endowment $e_i = (e_{i1}, \ldots, e_{in}) \in \mathcal{X}$. We denote this economy by $\mathcal{E}(u, e)$, where $u = (u_1, \ldots, u_m)$ and $e = (e_1, \ldots, e_m)$.

Our subsequent analysis will be greatly simplified in the case where for each agent one can distinguish consumption goods, those the agent consumes, from endowment goods, those with which the agent is endowed.[2] For agent $i \in A$, the set of endowment goods is given by $E_i = \{g \in G \mid e_{ig} > 0\}$. The set of consumption goods is denoted by C_i and characterized by the following assumption.

Assumption 1 (*Goods*)**.** For all $i \in A$, there exists $C_i \subset G$ such that $C_i \cap E_i = \emptyset$ and for all $x, y \in \mathcal{X}$ one has: $(\forall g \in C_i,\ x_g = y_g) \Rightarrow u_i(x) = u_i(y)$.

Accordingly, we define the set of *buyers* of good g as $B_g = \{i \in \mathcal{A} \mid g \in C_i\}$, and the set of *sellers* of good g as $S_g = \{i \in \mathcal{A} \mid g \in E_i\}$.

The following concepts are standard in the analysis of exchange economies.

- An allocation $x \in \mathcal{X}^m$ of goods is *feasible* if for all $g \in G$:

$$\sum_{i=1}^{m} x_{ig} \leq \sum_{i=1}^{m} e_{ig}.$$

We write $\mathcal{A}(e_1, \ldots, e_m) = \mathcal{A}(e) \subset \mathcal{X}^m$ for the set of feasible allocations.

[1] There are also notable exceptions such as Anderson et al. (2004), who show that continuous double auction can yield tâtonnement-like orbits in the Scarf economy.

[2] This setting is sometimes referred to as "buy-or-sell game" in the economic literature and is known under the name of Fisher economies in the algorithmic game theory literature.

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