Journal of Mathematical Economics 59 (2015) 24–36

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

Regular economies with ambiguity aversion[☆]

Noé Biheng, Jean-Marc Bonnisseau [*]

Paris School of Economics, Université Paris 1 Panthéon Sorbonne, 106-112 boulevard de l'Hôpital 75647, Paris Cedex 13, France

ARTICLE INFO

Article history:
Received 9 June 2014
Received in revised form
15 February 2015
Accepted 27 April 2015
Available online 8 May 2015

Keywords:
Demand function
General equilibrium
Ambiguity aversion
Multiprior preferences
Regular economies
Lipschitz behavior

ABSTRACT

We consider a family of exchange economies with complete markets where consumers have multiprior preferences representing their ambiguity aversion. Under a linear independence assumption, we prove that regular economies are generic. Regular economies exhibit enjoyable properties: odd finite number of equilibrium prices, local constancy of this number, local differentiable selections of the equilibrium prices.

Thus, even if ambiguity aversion is represented by non-differentiable multiprior preferences, economies retain generically the properties of the differentiable approach.

1. Introduction

Classically, the global analysis of the general economic equilibrium is based upon well known differential techniques. Basically, one requires the differentiability of the demand functions. We refer the reader to Debreu (1970), Mas-Colell (1985) and Balasko (1988) for more details.

This differentiability is often derived from well known assumptions on the utility functions. Indeed, the utility functions are supposed to be C^2 to obtain C^1 demand functions. This does not allow the presence of kinks on indifference curves that arise in uncertainty context.

In the maxmin expected utility model due to Gilboa and Schmeidler (1989), the agents face ambiguity modeled by the multiplicity of the priors of the agents. Each agent considers the minimum expected utility over his set of priors. This "minimum" generates kinks on the indifference curves when more than one probability realize the minimum, this leads to the non-differentiability of the demand functions. These kinks cannot be removed since they are genuinely linked to uncertainty not to modeling issues. The main objective of this paper is to get the genericity of regular economies despite that the demand functions are non-differentiable.

In this paper, we consider an exchange economy with a finite number ℓ of commodities and a finite number m of consumers. The preferences of consumer i are represented by a utility function u_i from $\mathbb{R}^{\ell}_{++}$ to $\mathbb{R}$. The function u_i is the minimum of a finite number n_i of expected utility functions that satisfy the usual differentiability requirements and a linear independence assumption on the extremal priors. For example, this linear independence assumption is satisfied by ε-contamination of confidence.

We first study the properties of the demand functions. This systematic study constitutes in itself a new result concerning consumers with multiprior preferences. Indeed, we prove that the demand functions are locally Lipschitz continuous and that these functions are continuously differentiable on an open set of full Lebesgue measure.

In the second part of the paper, we follow Balasko's program. We define and parametrize the equilibrium manifold. We show that it is indeed a smooth manifold at almost every point. As in the classical case, we can propose a global parametrization from which we deduce that the equilibrium manifold is lipeomorphic[1]

☆ This work was supported by the French National Research Agency, through the program Investissements d'Avenir, ANR-10–LABX-93-01. We wish to thank A. Chateauneuf for his corrections and his valuable comments and an anonymous referee for his remarks and the questions that he raised. All remaining errors are ours.

* Corresponding author.
E-mail addresses: Noe.Biheng@univ-paris1.fr (N. Biheng), Jean-marc.Bonnisseau@univ-paris1.fr (J.-M. Bonnisseau).

1 Two sets are *lipeomorphic* if there exists a one-to-one, onto and locally Lipschitz continuous mapping from the first set to the second one with a locally Lipschitz continuous inverse.

http://dx.doi.org/10.1016/j.jmateco.2015.04.004
0304-4068/

to an open connected subset of an Euclidean space denoted by $\mathcal{U}$ using similar approach than Bonnisseau and Rivera-Cayupi (2006).

We can define an extended projection using the parametrization. This mapping is continuously differentiable almost everywhere and locally Lipschitz continuous.

Contrary to the classical case, we have to take into account the kinks to define regular economies. A singular economy is either the image of a point where the extended projection is not differentiable or the image of a point where the differential mapping is not onto. A regular economy is, by definition, an economy that is not singular. By Sard's Theorem since the set $\mathcal{U}$ and the space of economies are two manifolds of same dimension, the set of singular economies is a set of Lebesgue measure zero.[2] By the Implicit Function Theorem, each regular economy has a finite number of equilibria and, around a regular economy, there exist continuously differentiable selections of the equilibrium prices.

Computing the degree of the extended projection by an homotopy argument, we obtain that every regular economy has a finite odd number of equilibrium prices.

We now mention earlier contributions. Rader (1973) showed that, when the consumers have demand functions almost everywhere differentiable satisfying property (N): "*The image of a null set is a null set.*", almost every economy has a finite number of equilibrium prices. In our paper, we prove that Rader's properties are satisfied by multiprior preferences but we get more with the local continuously differentiable selections.

Rigotti and Shannon (2012) study market implications of the presence of ambiguity modeled by variational preferences. Variational preferences encompass multiprior preferences. They show that almost all economies are determinate which means that there exist a finite number of equilibrium prices and local continuous selections. They obtain also a Lipschitz behavior in the Choquet case. Note that regularity and determinacy are two distinct concepts, the first one implying the second one. In particular, the number of equilibria may not be constant around a determinate economy. We need the linear independence of Assumption 1 to get regularity instead of determinacy.

In Dana (2004), Dana studies agents that are Choquet expected-utility maximizers. She is interested in equilibrium welfare properties and indeterminacy of the equilibrium. She provides a sufficient condition on equilibrium implying that there exists a continuum of equilibrium prices. But, she does not address the issue of genericity.

In Bonnisseau and Rivera-Cayupi (2006), Bonnisseau and Rivera-Cayupi study a non-smooth model although the failure of differentiability was not in the utility function but on the boundary of the consumption sets. They obtain demand functions with properties similar to ours.

In Section 2, we present multiprior preferences and the definition of an equilibrium with complete markets. Actually, to simplify the notation, we consider a larger class of preferences where the utility functions are defined as a minimum of a finite family of functions satisfying the usual differentiability requirements and a linear independence assumption on the gradient vectors. In Section 3, we study extensively the demand function of a consumer with multiprior preferences. The fourth section is devoted to the global analysis of the equilibrium manifold and to the genericity analysis. Some concluding remarks are given in Section 5 and finally, some technical proofs are given in Appendix.

[2] Actually, we also use that the image of a null set by a Lipschitz mapping is a null set.

2. Multiprior preferences

We[3] study a two-period economy with a complete system of markets. There are two dates $t = 0$ and $t = 1$. There is uncertainty at date 0 about which state will occur at date 1. At date 1, there are S states of nature. We denote by $\Delta(S)$ the set of probabilities on $\mathcal{S} = \{1, \dots, S\}$. There are I goods at each node so there are $\ell := I(1+S)$ goods. We model the ambiguity by a multiplicity of probabilities.

From a general equilibrium point of view, we study a family of economies parametrized by strictly positive endowments with m consumers and ℓ commodities. We denote respectively by M and L the set of consumers and the set of commodities. Let $M \equiv \{1, \dots, m\}$ and $L \equiv \{1, \dots, \ell\}$.

For each agent $i \in M$, there exists a closed convex set $\mathcal{P}^i \subset \Delta(S)$. We suppose that the set $\mathcal{P}^i$ has n_i extremal points $(\pi_i^k)_{1\le k\le n_i}$. We also suppose that the set $\mathcal{P}^i$ is contained in $\mathbb{R}^S_{++}$ to get the strict monotony of preferences. This can in particular correspond to the convex case of the C.E.U. (Choquet Expected Utility) model of Schmeidler (1989) since the core of a convex capacity has at most $S!$ extremal points (Shapley, 1989).

The agent chooses a contingent consumption vector $(x_s)_{1\le s\le S} \in \mathbb{R}^{IS}_{++}$ and a vector $x_0 \in \mathbb{R}^I_{++}$ corresponding to his consumption at date zero. The utility of the agent i is given by:

$$\begin{aligned} u_i(x) &= b_{i0}(x_0) + \min_{\pi_i \in \mathcal{P}^i} \sum_{s=1}^{S} \pi_i(s) b_{is}(x_s) \\ &= \min_{1\le k\le n_i} \left\{ b_{i0}(x_0) + \sum_{s=1}^{S} \pi_i^k(s) b_{is}(x_s) \right\} \end{aligned} \tag{2.1}$$

where $b_{is} : \mathbb{R}^I_{++} \longrightarrow \mathbb{R}$ are $1+S$ functions. We define, for $k \in \{1, \dots, n_i\}$, the function u_i^k by:

$$u_i^k(x) = b_{i0}(x_0) + \sum_{s=1}^{S} \pi_i^k(s) b_{is}(x_s) \quad \text{for } x \in \mathbb{R}^{(1+I)S}_{++}.$$

In many applications, the function b_{is} does not depend on the state s. The state-dependent case in the expected utility model has been studied by Karni et al. (1983) and Wakker (1987) for example. For more references and a presentation of some applications, see Karni (1985).

We posit the following assumption on the probability vectors $(\pi_i^k)_{1\le k\le n_i}$.

Assumption 1. For every $i \in M$, the probability vectors $(\pi_i^k)_{1\le k\le n_i}$ are linearly independent.

Note that this assumption holds true when $\mathcal{P}^i$ is an ε-contamination of a probability $\bar\pi$. Recall that a set $\mathcal{P}$ is called an ε-contamination if: $\mathcal{P} := (1-\varepsilon)\{\bar\pi\} + \varepsilon\Delta(S)$ for some real number $\varepsilon \in]0; 1[$. The extremal points of $\mathcal{P}$ are $(1-\varepsilon)\bar\pi + \varepsilon\pi^s$ for $s = 1, \dots, S$, where π^s is the probability such that $\pi^s(s) = 1$. Obviously, these vectors are linearly independent. The ε-contamination of confidence is a special case of the Choquet Expected Utility model. Indeed, the related capacity ν is defined by:

$$\nu(A) := \begin{cases} (1-\varepsilon)\bar\pi(A) & \text{if } A \neq \mathcal{S} \\ 1 & \text{if } A = \mathcal{S} \end{cases}$$

[3] Notations. If x is a vector of $\mathbb{R}^\ell$, the norm of the vector x is defined by $\|x\| := \sum_{h=1}^{\ell} |x_h|$. The left-derivative of a function defined on an open interval $J \subset \mathbb{R}$ at $t \in J$ is denoted by $\Psi'^l(t)$. Similarly the right-derivative of Ψ at t is denoted by $\Psi'^r(t)$. The vector $\mathbf{1}$ denotes the vector of $\mathbb{R}^\ell$ that has all coordinates equal to one. The inner product of x and y elements of $\mathbb{R}^\ell$ is: $x \cdot y := \sum_{h=1}^{\ell} x_h y_h$. For all $r > 0$, $B_o(a, r)$ (respectively $B_c(a, r)$) denotes the open (resp. closed) ball of center a and of radius r. $\sharp K$ denotes the cardinal of the set K. The vectors are, by convention, column vectors and the transpose of a vector x is denoted by x^T.