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Wealth concerns and equilibrium☆

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ABSTRACT

Most properties of the classical general equilibrium model without externalities fail to extend to the wildest forms of consumption externalities. The recent interest for wealth concerns, a kind of externality associated with herding behavior and other-regarding preferences, motivates a study of the general equilibrium exchange model with those externalities. The diffeomorphism of the equilibrium manifold with a Euclidean space, the smoothness and properness of the natural projection and its non-zero degrees are shown to hold true for endowment spaces with variable total resources. Other properties of the classical exchange model without externalities are fragile in the sense that they do not resist the introduction of wealth concerns even in models where consumers preferences are represented by the simplest forms of utility functions like the log-linear (or Cobb–Douglas) functions. The most notable fragile properties are the uniqueness and regularity of equilibrium at equilibrium allocations and the stability of no-trade equilibria.

1. Introduction

The first discussions of externalities to appear in book form go back to Sidgwick (1901), Marshall (1920) and Veblen (1899). This early recognition of the importance of externalities in consumption and production is to be contrasted with the relatively small number of theoretical results that are currently available. With preferences that can be represented by separable utility functions that are the sum of a direct utility for the consumption of goods and of another function that represents the impact of externalities as in Dufwenberg et al. (2011), equilibrium prices and allocations are those of the economy without externalities defined by the utility functions for goods and the classical existence and regularity properties directly apply to that setup. Without separability, the existence of equilibrium for economies that are convex once externalities are fixed has been proved, but only recently, by del Mercato (2006), del Mercato and Platino (2011), and Ericson and Kungy (2012). Still without separability, regular economies fail to be generic despite an equal number of independent equilibrium equations and unknowns as follows from an example of Bonnisseau and del Mercato (2010). Kung succeeds however in proving the genericity of regular equilibria for parameter spaces that are big enough to include preferences and endowments (Kung, 2008). Whereas, Bonnisseau and del Mercato show that, without any need for the perturbations of the preferences, the genericity of regular equilibria holds true as long as the second-order external effects on the individual preferences are not too strong along the equilibrium budget lines (Bonnisseau and del Mercato, 2010).

These results leave open the question of whether other properties of the classical model without externalities are satisfied by models where externalities are somewhat mild (when compared to the most extreme forms of externalities) but already too large to be assimilated to small perturbations. In that direction, the externalities that result from consumers having preferences that also depend on the wealth of the other consumers in the economy have captured the interest of economists in the last two decades. The interest for these forms of externalities, also known as wealth concerns, can be traced back to the publication of Frank's book on the role of relative wealth concerns in the determination of social status (Frank, 1985). Research has then been particularly active in two fields that ordinarily have little in common. In Finance, Abel and Gali explain herding behavior by introducing wealth concerns in asset pricing models (Abel, 1990; Gali, 1994). A more recent article by DeMarzo, Kaniel and Kremer also sees rational bubbles as a consequence of wealth concerns (DeMarzo et al., 2007). In Public Economics, the importance of wealth concerns is emphasized through the concepts of "other-regarding" or "social" preferences by Fehr, Gächter and Schmidt to name a few (Fehr and Gächter, 2000; Fehr and Schmidt, 2006). The first study of wealth concerns in general equilibrium models is due to Dufwenberg et al. who show that the two theorems of welfare economics do not hold true under wealth

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concerns even when consumers' preferences can be represented by utility functions that are the sum of a standard direct utility for goods consumption and another term that represents the impact, positive or negative, of the distribution of wealth throughout the economy (Dufwenberg et al., 2011).

The focus is placed in this paper on the relationships between endowments and associated equilibrium allocations in general equilibrium models with wealth externalities. The goal is then to identify what properties resist the introduction of wealth externalities. Consumers are represented by demand functions that depend on prices and on consumers' wealth without necessarily resulting from the budget constrained maximization of some utility functions. That level of generality is justified by potential future applications to models of asset pricing.

The study of the properties of the general equilibrium model with wealth concerns proceeds by following the equilibrium manifold and natural projection approach developed in Balasko (1975). See Balasko (2011) for a recent exposition in book form. It is then quite remarkable that the equilibrium manifold over a parameter space that consists of initial endowments only has, under wealth concerns, essentially the same structure as in the classical case without externalities. This structure is the one of a smooth manifold made of linear fibers parameterized by the no-trade equilibria. It follows readily from this structure that, as in the classical case without externalities, the equilibrium manifold is diffeomorphic to a Euclidean space, an essential property for comparative statics, and also that the natural projection is a ramified proper covering of the endowment set. In economic terms, immediate byproducts of that structure are the existence of equilibria for all economies and the finiteness of the number of equilibria at regular economies, the smooth dependence of regular equilibria on endowments and the genericity of regular economies for variable total resources. More importantly, the picture of the equilibrium manifold over the endowment set is the same with and without wealth concerns.

Not all properties of the classical model without externalities extend to wealth concerns. The following properties of the classical model without externalities are fragile in the sense that they do not resist the introduction of sufficiently large wealth concerns: the regularity of equilibrium allocations; the uniqueness of equilibrium at equilibrium allocations; the stability of no-trade equilibria. I show in this paper that these properties already fail to be satisfied in models with log-linear (or Cobb–Douglas) utility functions with wealth concerns.

This paper is organized as follows. The main assumptions, definitions and notation occupy Section 2. Section 3 deals with the equilibrium manifold and proves that it is a smooth manifold diffeomorphic to a Euclidean space. Section 4 proves that the natural projection is a smooth and proper map, properties from which a large number of properties of the general equilibrium model with wealth concerns can be derived. Section 5 is devoted to showing that the uniqueness, regularity and stability of equilibrium at endowments that are already equilibrium allocations is not always satisfied when consumers do have wealth concerns. The conclusion occupies Section 6. Lengthy but elementary proofs of several intermediary properties are relegated to Appendix A. Appendix B contains a quick presentation of the most important mathematical concepts required by the study of smooth mappings and their singularities. With the help of that Appendix, familiarity with differential topology is not necessary.

2. Assumptions, definitions and notation

This section describes the main assumptions regarding goods, prices, consumers' preferences and endowments.

2.1. Goods and prices

There is a finite number ℓ of goods and the commodity space is $\mathbb{R}^\ell$. Let $z = (z^1, \dots, z^\ell) \in \mathbb{R}^\ell$ be a commodity bundle. The vector $\bar{z} = (z^1, \dots, z^{\ell-1}) \in \mathbb{R}^{\ell-1}$ is obtained from z by deleting the last coordinate z^ℓ.

Prices are strictly positive. The price vector $p = (p_1, \dots, p_\ell) \in X$ is normalized by the numeraire convention where the ℓ-th good is the numeraire. This is equivalent to setting $p_\ell = 1$. Let $\mathbb{S} = \mathbb{R}^{\ell-1}_{++} \times \{1\}$ denote the set of numeraire normalized prices. The numeraire normalized price vector $p \in \mathbb{S}$ can also be written as $p = (\bar{p}, 1)$ where $\bar{p} = (p_1, \dots, p_{\ell-1}) \in \mathbb{R}^{\ell-1}_{++}$ expresses the prices of the non-numeraire goods.

We will need properties of consumer's demand when the price of the numeraire tends to zero relatively to the prices of some non-numeraire goods. This is equivalent to the numeraire normalized prices of some non-numeraire goods tending to $+\infty$. Such issues are easily handled by the use of homogeneous or projective coordinates. By definition, the homogeneous coordinates $(p_1, p_2, \dots, p_\ell)$ and $(\alpha p_1, \alpha p_2, \dots, \alpha p_\ell)$, where $\alpha \neq 0$ is a real number, represent the same point of the projective space $P(\mathbb{R}^\ell)$. That projective space can be identified to the set of lines of $\mathbb{R}^\ell$ that pass through the origin. The lines that intersect the hyperplane defined by equation $p_\ell = 1$ can then be identified with their intersection point (with that hyperplane). The projective space $P(\mathbb{R}^\ell)$ is then the disjoint union of the hyperplane $p_\ell = 1$ and the set that consists of the lines through the origin that are contained in the hyperplane defined by equation $p_\ell = 0$. Those lines are interpreted as "points at infinity" of the hyperplane $p_\ell = 1$. Let $P(\mathbb{S})$ be the closure of the subset of the projective space $P(\mathbb{R}^\ell)$ that is generated by the points of the set of numeraire normalized prices $\mathbb{S}$. Then, $P(\mathbb{S})$ can be viewed as the disjoint union of $\mathbb{R}^{\ell-1}_{+} \times \{1\} = \{p = (p_1, \dots, p_{\ell-1}, p_\ell) \mid p_j \geq 0, \text{ and } p_\ell = 1\}$ (note that some prices can now be equal to 0) and the set of non-negative directions of $\mathbb{R}^{\ell-1}_{+}$ that correspond to the points at infinity of $P(\mathbb{S})$. The projective space $P(\mathbb{R}^\ell)$ is compact, from which follows that its closed subset $P(\mathbb{S})$ is also compact.

2.2. Endowments and wealth

An exchange economy consists of m consumers. The consumption space of every consumer is the strictly positive orthant $X = \mathbb{R}^\ell_{++}$. Consumer i, with $1 \leq i \leq m$, is endowed with a commodity bundle $\omega_i \in X$. Given the price vector $p \in \mathbb{S}$, consumer i's wealth w_i is the inner product $w_i = p \cdot \omega_i$. Wealth w_i is a strictly positive number for any pair $(p, \omega_i) \in \mathbb{S} \times X$.

Let $\Omega = X^m$, the m-time Cartesian product of X. The m-tuple $\omega = (\omega_1, \dots, \omega_m) \in \Omega = X^m$ represents the endowments in goods of all the consumers in the economy. The set of price–wealth vectors is denoted by $\mathbb{B} = \mathbb{S} \times \mathbb{R}^m_{++}$. This set is diffeomorphic to $\mathbb{R}^{\ell+m-1}$. The vector $b = (p, w_1, \dots, w_i, \dots, w_m) \in \mathbb{S} \times \mathbb{R}^m_{++}$ describes at once the prices of the ℓ goods and the wealth of every consumer in the economy.

The vector obtained from $\omega = (\omega_1, \dots, \omega_m) \in \Omega$ by deleting the ith component ω_i is denoted by $\omega_{-i} = (\omega_1, \dots, \omega_{i-1}, \omega_{i+1}, \dots, \omega_m) \in X^{m-1}$. Similarly, $\bar{\omega}_{-i}$ (resp. w_{-i}) is obtained from $\bar{\omega} = (\bar{\omega}_1, \dots, \bar{\omega}_m)$ (resp. $w = (w_1, \dots, w_m)$) by deleting the ith component $\bar{\omega}_i$ (resp. w_i).

Remark 1. Simpler mathematics and the proximity of numeraire with money are the usual justifications for normalizing prices by way of a numeraire good whose price is, by definition, equal to one. In classical consumer theory with no externalities, utility maximization subject to a budget constraint yields demand functions of price and wealth that are homogeneous of degree zero, a property that is interpreted as the absence of monetary illusion. Then, the choice of the numeraire and more generally of