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# Inflationary equilibrium in a stochastic economy with independent agents

John Geanakoplos [a,b,*], Ioannis Karatzas [c,d], Martin Shubik [a,b], William D. Sudderth [e]

[a] Yale University, Box 208281, New Haven, CT 06520-8281, United States
[b] Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, United States
[c] Columbia University, MC 4438, New York, NY 10027, United States
[d] INTECH Investment Management, One Palmer Square, Suite 441, Princeton, NJ 08542, United States
[e] University of Minnesota, 224 Church Street SE, Minneapolis, MN 55455, United States

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ABSTRACT

We prove the existence of stationary monetary equilibrium with inflation in a "Bewley" model with constant aggregate real variables but with idiosyncratic shocks to the endowments of a continuum of individual agents, when a central bank stands ready to borrow or lend fiat money at a fixed nominal rate of interest and the agents face borrowing constraints. We also find that, in the presence of real micro uncertainty about individual endowments, the rate of inflation is higher (equivalently, the real rate of interest is lower) than it would be in a "certainty-equivalent economy"; to wit, one in which every agent's endowment is replaced by its expected value. Thus, underlying microeconomic uncertainty and borrowing constraints are shown to generate additional inflation.



## 1. Introduction

We seek to understand the behavior of prices and money in a simple infinite-horizon economy with a central bank and one nondurable commodity. Following Bewley (1986) we consider an economy in which a continuum of agents are subject to idiosyncratic, independent and identically distributed random shocks to their endowments. At the micro level the economy is in perpetual flux but, at the macro level, aggregate endowments remain constant across time and states. We prove the existence of a stationary equilibrium that also remains rock-steady at the macro level despite micro turmoil in individual consumption and saving. Stationary equilibrium means that markets clear, and prices and money grow at a deterministic rate $\tau$, all the while maintaining the same distribution of real (inflation-corrected) wealth across agents. In each period some formerly rich agents may become poor, and vice versa, but the fraction of the population at every level of real wealth remains the same.

Bewley proved the existence of a stationary equilibrium in a more general economy than ours, allowing for example for multiple commodities.[1] But his model did not have a central bank that could change the supply of money over time, and therefore had no inflation in equilibrium. Inflation seems to complicate the question of existence of equilibrium. We are not aware of any other existence proof for stationary equilibrium with inflation in a Bewley-style model.

On the other hand, there is a large literature on a similar kind of model without money, but with a capital sector that can be used to produce output. Huggett (1993) and Aiyagari (1994) prove the existence of stationary equilibrium.[2] Our method of proof uses many of the same elements: we invoke properties of the dynamic programming problem just as they did, and then we analyze a fixed point problem involving the real rate of interest (or equivalently the rate of inflation) much like they did. More recently Miao (2002)

* Corresponding author at: Yale University, Box 208281 New Haven, CT 06520-8281, United States.
*E-mail addresses:* john.geanakoplos@yale.edu (J. Geanakoplos), ik@math.columbia.edu, ik@enhanced.com (I. Karatzas), martin.shubik@yale.edu (M. Shubik), bill@stat.umn.edu (W.D. Sudderth).

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[1] Bewley also allowed for Markovian random endowments and for heterogeneous utility functions. All of these extensions could probably be accommodated in our setting as well.

[2] Huggett's proof is for the special case where endowments can take on only two values and utility is given by the functional form $u(x) = x^\alpha$. Aiyagari states his existence theorem, but the proof appears in an appendix that was not published. The working paper version of the proof is missing some details.

and Kuhn (2013) have given existence proofs in similar kinds of models based on lattice theory. The details of our proof are different, and we use different assumptions on the utilities. There is no fiat money in these other models.

Huggett (1997) and Aiyagari (1994), like Laitner (1979, 1992), Bewley (1986), and Clarida (1990) before them in somewhat different models, prove that the stationary real rate of interest is below the time discounting of the agents, without invoking any assumption on the third derivative of the utilities.[3] The cause is the constraint on borrowing. Ljungqvist and Sargent (2000) survey these papers. We obtain analogous results for our model, which differs in having fiat money with inflation.

In this paper we study a model with a continuum of agents with a common discount rate $\beta$ and common instantaneous utility function $u(\cdot)$, but with idiosyncratic shocks to their endowments that leave the aggregate endowment constant. For such a model, and without borrowing or lending, it is already known that there exists in great generality an equilibrium with a stationary distribution of nominal wealth and a constant commodity price; cf. Karatzas et al. (1994). Bewley (1986) showed that such a noninflationary stationary equilibrium also exists when there is borrowing and lending but at a zero rate of interest. We confirm this result in Section 7.6.

We add to the model a central bank committed to borrowing or lending with every agent at a fixed nominal interest rate $\rho > 0$. After the recent changes at the Fed instituted by chairman Bernanke, this corresponds to the ability of the American central bank to pay interest on deposits as well as to receive interest on loans. We do not add a Treasury to the model; we simply allow the central bank to print as much money as it needs to in order to pay depositors' interest, or to retire as much money as it receives from interest payments it receives. We also assume a cash-in-advance constraint, so that all individual purchases of goods must be paid for by cash. In the Bewley (1986) model, agents could purchase commodities by using the revenue obtained by the simultaneous selling of other commodities; implicitly Bewley assumed a standing credit market at zero rate of interest. In our model agents must sell their entire endowment for cash, while simultaneously buying goods for cash (perhaps borrowed, but at a rate $\rho > 0$). One interpretation, similar to that used by Lucas, is that the productive and consumption arms of each agent act separately.[4] This sell-all assumption makes the existence of monetary equilibrium easier to demonstrate. Nevertheless, with a central bank fixing a positive rate of interest, a noninflationary equilibrium rarely exists. (A necessary condition for existence is that the bank selects an interest rate that "balances the books" so that all the lending comes from one agent to another and the aggregate money supply remains constant; see Karatzas et al., 1997 and Geanakoplos et al., 2000.) In an inflationary equilibrium, the supply of outside money that agents own free and clear of any obligations at the beginning of each period must change over time. This non-existence of stationary equilibrium creates the added complication in our models compared to the rest of the Bewley-style literature.

We prove here the existence of stationary inflation-corrected equilibrium, under certain technical conditions and under a critical borrowing constraint (Theorem 7.1), for any $\rho > 0$. More specifically, we assume that all agents have a strictly concave utility function $u(\cdot)$ whose derivative is bounded away from zero. Another important assumption is that agents can only borrow up to a fraction $\theta$ of the discounted value of their current endowment. As long as $\theta \le 1$, we have a model of lending secured by future income and without any chance of default around equilibrium. We were not able to establish existence in general with $\theta = 1$, though we show that such an equilibrium does exist in the absence of microeconomic uncertainty (Example 6.1). Instead we prove the weaker result that stationary inflation-corrected equilibrium exists whenever $0 \le \theta \le \theta^*(\rho)$, where the upper bound $\theta^*(\rho) \in (0, 1)$ decreases as $\rho$ increases. The need for such an upper bound illustrates the difference between our model and the previous Bewley, Huggett, and Aiyagari models.

The existence of inflation-corrected equilibrium allows us to study the effect of micro uncertainty and of the borrowing constraint on the rate of inflation and on the real rate of interest. In a world of micro certainty, which we could obtain in our setting by replacing each individual agent's random endowment with its expected value, the rate of inflation $\tau$ would necessarily satisfy the famous Fisher equation

$$\tau = \beta(1 + \rho),$$

provided $\theta = 1$. The Fisher equation also holds in our model, even with uncertainty, if the equilibrium is interior; that is, if agents never forgo consumption and if they are never forced by the collateral constraint to borrow less than they would like (Theorem 5.1).

We prove, however, that if $\theta \le \theta^*(\rho)$, then there is always an equilibrium in which $\tau > \beta(1 + \rho)$, whether or not there is micro uncertainty and no matter what the sign of $u'''(\cdot)$ (Theorem 7.1). Our paper thus establishes the principle that borrowing constraints generate additional inflation beyond what would be predicted by the central bank rate of interest and the discount rate of the agents.

We prove that if there is genuine micro uncertainty, and if the marginal utility function $u'(\cdot)$ is strictly convex, then all stationary equilibria have $\tau > \beta(1 + \rho)$, irrespective of the bound $\theta$ on borrowing. Thus, with genuine randomness in the endowments and with $u'(\cdot)$ strictly convex, stationary equilibrium can only exist when a non-negligible fraction of the agents is up against their borrowing constraints (Theorem 5.2). Thus micro uncertainty and borrowing constraints increase the rate of inflation beyond what might be expected from the Fisher equation.

We can also interpret our result in terms of the implied real rate of interest rather than in terms of the rate of inflation. Fisher defined the *real rate of interest* $\bar{\rho}$ by

$$1 + \bar{\rho} \equiv \frac{1 + \rho}{\tau}.$$

In our model with certainty and $\theta = 1$, the Fisher equation must hold; that is, the real rate of interest necessarily equals the reciprocal of the discount: $1/\beta = 1 + \bar{\rho}$. Our Theorems 7.1 and 5.2 show that, with genuine micro uncertainty, the real rate of interest will be less than the reciprocal of the time discount. This interpretation of our inflation principle shows its close resemblance to the results of Huggett (1997), Aiyagari (1994), Laitner (1979, 1992), Bewley (1986), and Clarida (1990), where it is typically assumed that the agents can trade a real bond that pays the same inflation-corrected amount in each future state.

In an earlier paper on this subject (Karatzas et al., 2006) we showed that macroeconomic uncertainty creates inflation. There we had a representative agent and random i.i.d. aggregate endowments. Prices necessarily jumped around from period to period, but we showed that, in stationary equilibrium, the long-run rate of inflation was always uniquely defined and higher than $\beta(1 + \rho)$. There we did not need to invoke a borrowing constraint more severe than necessary to rule out default. Taken together, our two papers provide a causal link between fluctuations in endowments (or production) and inflation.

A precise formulation of our model, and of equilibrium, is given in the next section. The notion of stationary equilibrium is defined

3 Huggett (1997) proves that all equilibria must have this property, while some of the other papers prove that at least one equilibrium must have this property.

4 Another interpretation is that no agent can eat his own endowment, but is indifferent to the goods of all others.